TECHNIQUES AND METHODS FOR PERTURBATIVE QCD

PSI Summer School on Particle Physics:
Gearing up for the LHC
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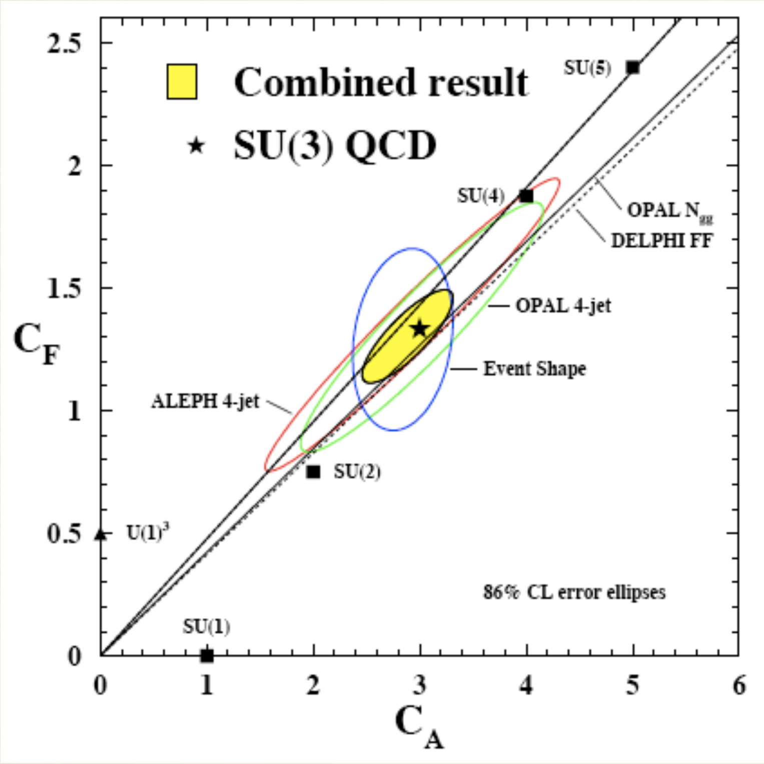
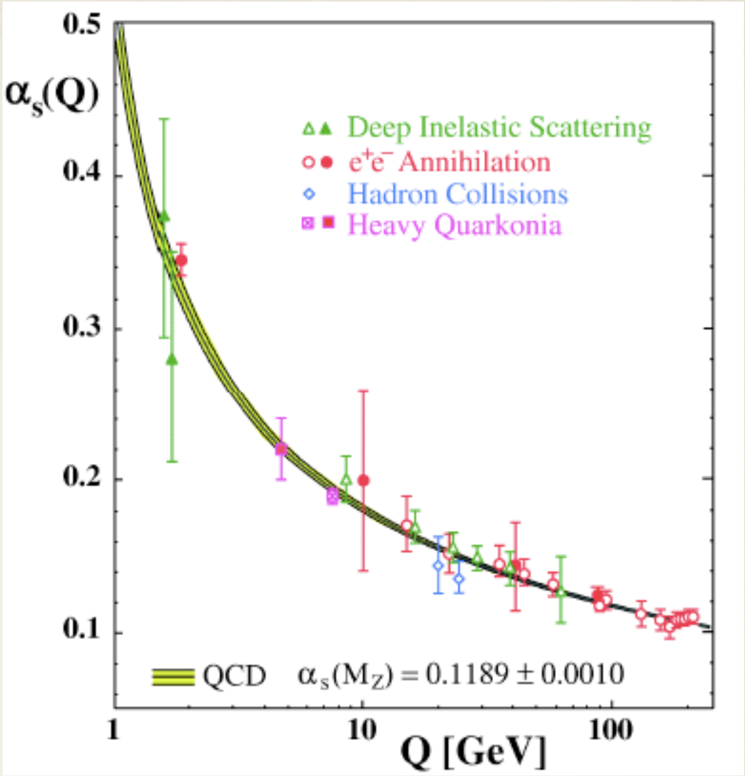
Outline

- Framework for QCD at colliders: factorization and universality; review of pQCD formulae
- Example #1: $e^+e^- \to$ hadrons at NLO; infrared singularities; the Sudakov form factor and the parton shower; jets
- Example #2: deep-inelastic scattering; factorization of IR singularities; DGLAP evolution; PDFs and their errors
- Example #3: Higgs production in gluon fusion; why NLO corrections can be large; effective field theory; integration-by-parts
- Survey of advanced topics at the LHC: matching fixed-order and parton showers; techniques for multi-leg LO and NLO; calculations at NNLO

Status of pQCD





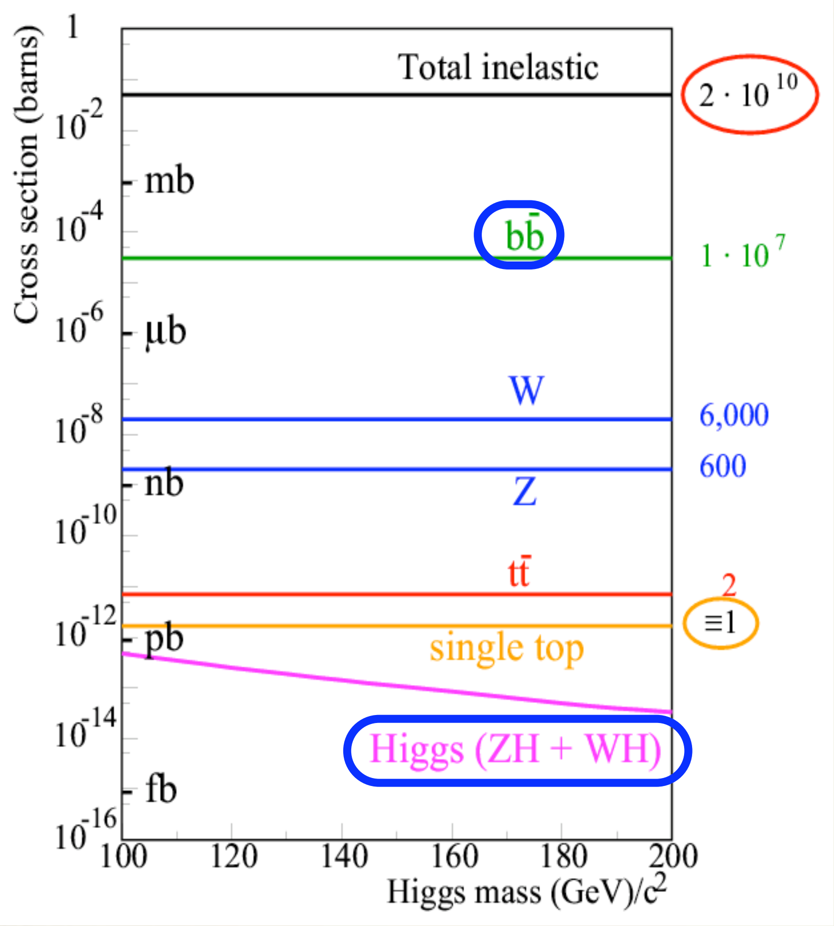
SU(3) gauge theory of QCD established as theory of Nature

Predicted running of α_s established in numerous experiments over several orders of magnitude

2004: Gross, Politzer, Wilczek

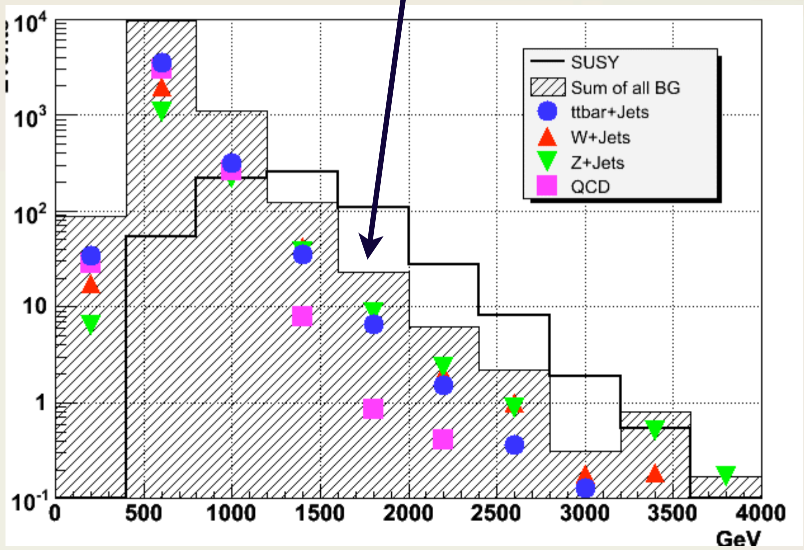
Why do we still care about QCD?

The revolution crushed

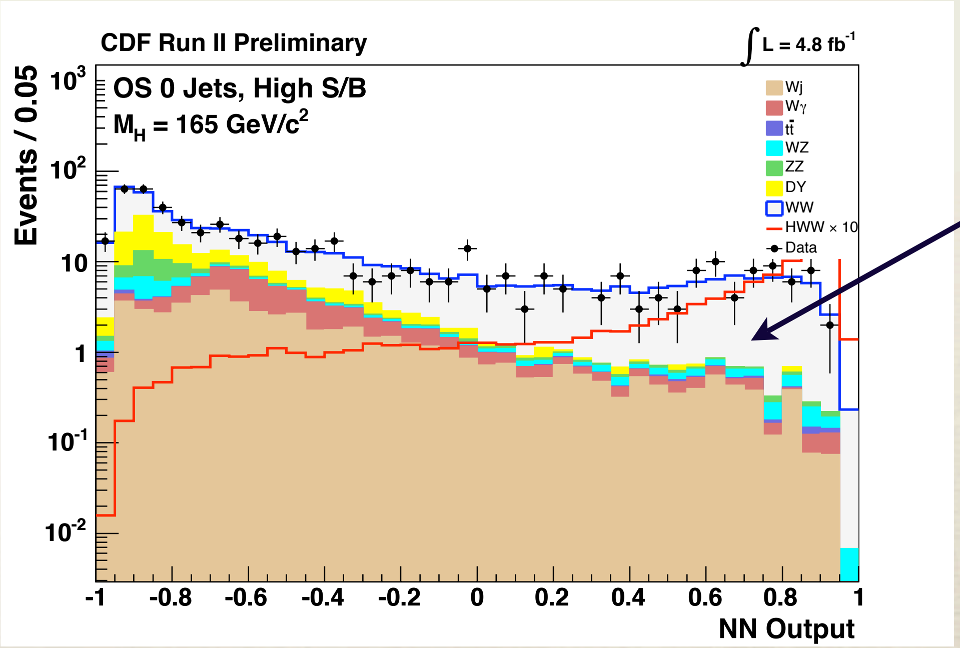


Enormous challenge to understand signal, background to be sure of discovery!

Do we understand the QCD shape prediction for W/Z+jets?

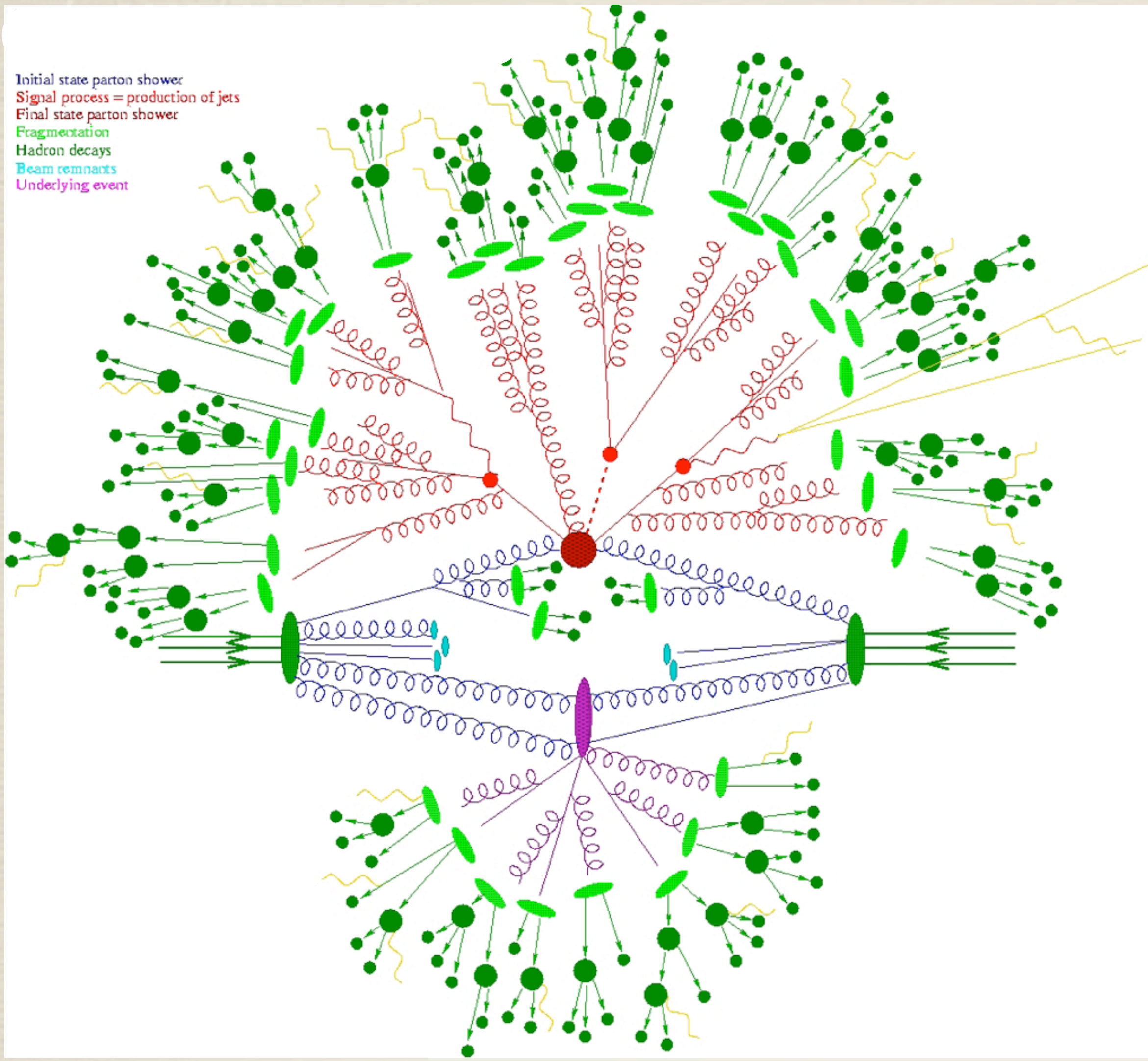


ATLAS TDR: S/B >10
Current: S/B~2



What is the QCD prediction for the di-boson production rate?

Collisions at the LHC



A lot going on...

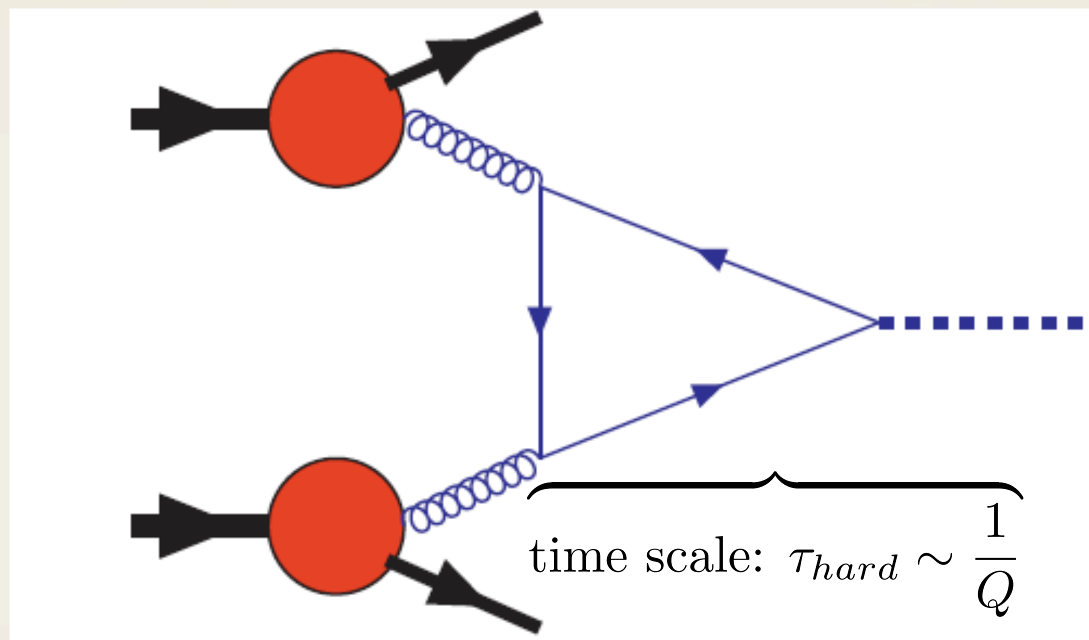
- New physics at hard scale; M_H for example
- Parton shower evolution from M_H to Λ_{QCD}
- Final state hadronization at Λ_{QCD}
- Parton distribution functions at Λ_{QCD}
- Multiple parton interactions, hadron decays, ...

How does one make a prediction for such an event?

Divide and conquer

Make sense of this with *factorization*: separate hard and soft scales

$$\underbrace{\text{time scale: } \tau_{proton} \sim \frac{1}{\Lambda_{QCD}}}$$



Review of factorization theorems: *Handbook of pQCD*, cteq.org; Collins, Soper, Sterman hep-ph/0409313

$$\sigma_{h_1 h_2 \to X} = \int dx_1 dx_2 \underbrace{f_{h_1/i}(x_1; \overbrace{\mu_F^2}^{\text{factorization scale}}) f_{h_1/j}(x_2; \mu_F^2)}_{PDFs} \underbrace{\sigma_{ij \to X}(x_1, x_2, \mu_F^2, \{q_k\})}_{\text{partonic cross section}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^n}_{\text{power corrections}}$$

Non-perturbative but *universal*; measure in DIS, fixed-target, apply to Tevatron, LHC

Process dependent but calculable in pQCD

Small for sufficiently inclusive observables

Recipe for a QCD prediction

- Calculate $\sigma_{ij \to X}$
- Evolve initial, final states to Λ_{QCD} using parton shower
- Connect initial state to PDFs, final state to hadronization

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How precisely must we know σ?

Do we know how to combine σ, parton shower?

Are our observables inclusive or must we worry about large logarithms?

Do we have hard jets? Parton showers assume soft/ collinear radiation

Do we know the PDFs in the relevant kinematic regions?

Field theory of QCD

- Review of the basic formulae from SU(3) gauge theory of QCD

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^A_{\alpha\beta}F^{\alpha\beta}_A + \sum_{flavors} \bar{q}_a \left(i\not{D} - m\right)_{ab} q + \mathcal{L}_{gauge} + \mathcal{L}_{ghost} \\ F^A_{\alpha\beta} &= \partial_\alpha A^A_\beta - \partial_\beta A^B_\alpha - g_s f^{ABC} A^B_\alpha A^C_\beta \\ (D_\alpha)_{ab} &= \partial_\alpha \delta_{ab} + i g_s t^C_{ab} A^C_\alpha\end{aligned}$$

anti-symmetric structure constants

strong coupling constant

a=1,...,3; quark in fundamental rep.

A=1,...,8; gluon in adjoint rep.

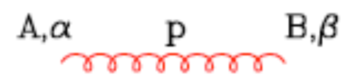
Basic group theory facts:

$$\begin{aligned}\mathrm{Tr} t^A t^B &= \frac{1}{2}\delta^{AB} \\ \sum_A \left(t^A t^A\right)_{ab} &= C_F \delta_{ab}, \quad C_F = \frac{N^2-1}{2N} = \frac{4}{3} \\ \sum_{A,B} f^{ABC} f^{ABD} &= C_A \delta^{CD}, \quad C_A = N = 3\end{aligned}$$

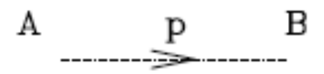
R_ξ gauge-fixing term:

$$\mathcal{L}_{gauge} = \frac{1}{2\lambda}\left(\partial_\alpha A^A_\alpha\right)^2$$

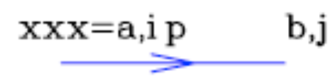
Feynman rules



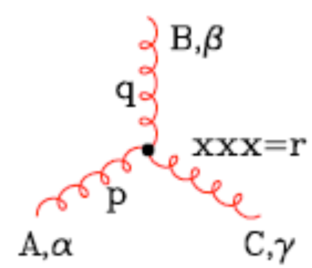
$$\delta^{AB}\ [-g^{\alpha\beta}+(1-\lambda)\,\frac{p^\alpha p^\beta}{p^2+i\epsilon}]\ \frac{i}{p^2+i\epsilon}$$



$$\delta^{AB}\ \frac{i}{(p^2+i\epsilon)}$$

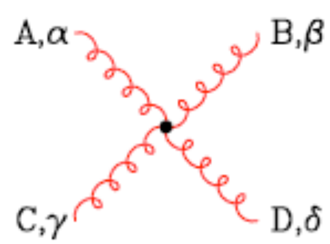


$$\delta^{ab}\ \frac{i}{(\not p-m+i\epsilon)_{ji}}$$

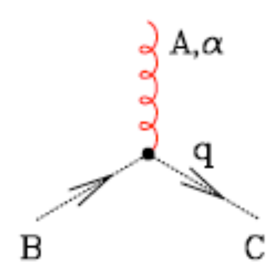


$$-g\ f^{ABC}[(p-q)^\gamma g^{\alpha\beta}+(q-r)^\alpha g^{\beta\gamma}+(r-p)^\beta g^{\gamma\alpha}]$$

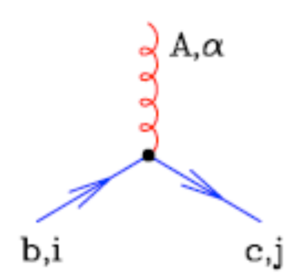
(all momenta incoming)



$$-ig^2\ f^{XAC}f^{XBD}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$
$$-ig^2\ f^{XAD}f^{XBC}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$$
$$-ig^2\ f^{XAB}f^{XCD}\ [g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$



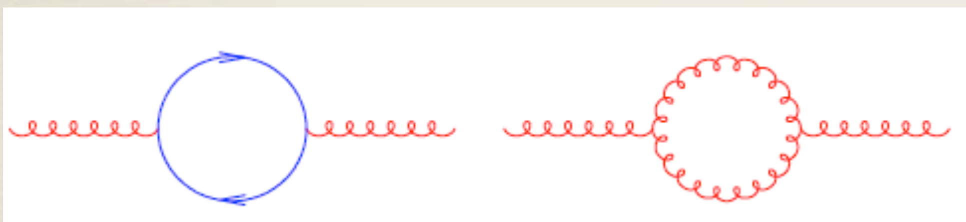
$$g\ f^{ABC}q^\alpha$$



$$-ig\ (t^A)_{cb}\ (\gamma^\alpha)_{ji}$$

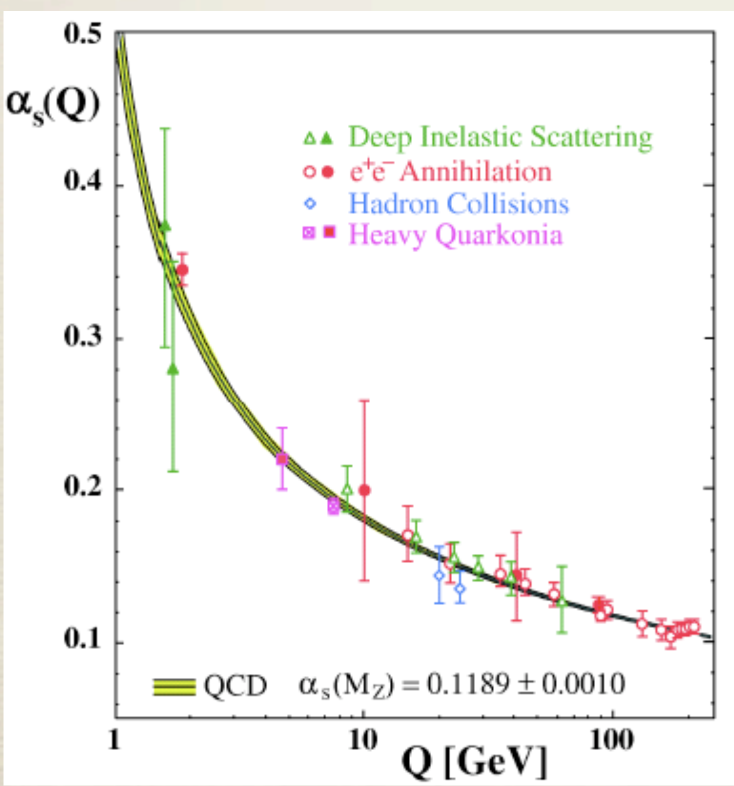
The running coupling

- Renormalization of ultraviolet divergences introduces a logarithmic dependence of $\alpha_s = g_s^2/4\pi$ on energy scale



$$\alpha_S(Q) = \frac{\alpha_S(\mu)}{1+\alpha_S(\mu) b\tau}\,, \quad \tau = \ln\left(\frac{Q^2}{\mu^2}\right)$$

$$b = \frac{(11C_A - 2N_f)}{12\pi}$$



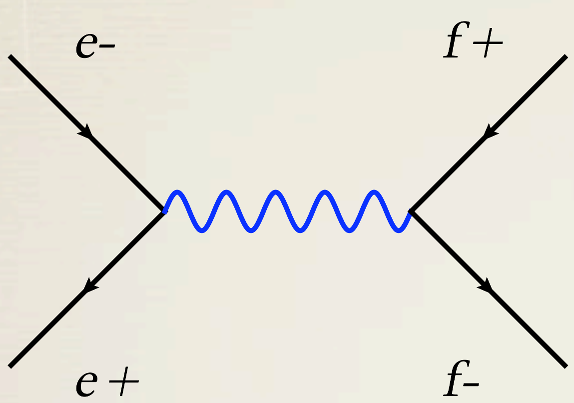
- *Asymptotic freedom*: small at high energies, we can compute in perturbation theory
- Coupling constant blows up at scale Λ_{QCD},
- Expect confinement, hadronization to occur at distances $L \sim 1/\Lambda_{QCD}$

$$\alpha_S(Q) = \frac{1}{b\ln(Q^2/\Lambda^2)}$$

Example 1: e^+e^- to hadrons at NLO

The basics: IR singularities in e^+e^-

- Many QCD issues relevant to hadronic collisions appear here.

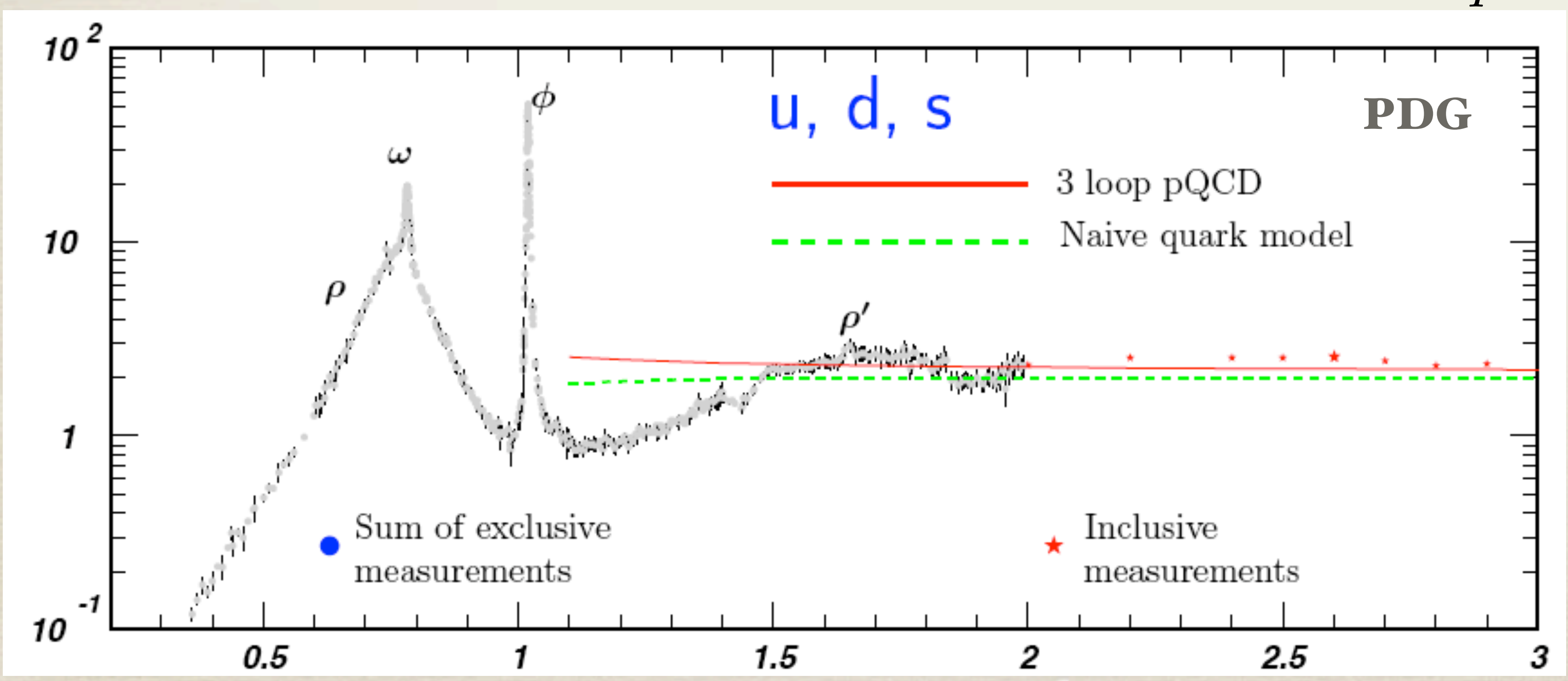


$$R = \frac{\sigma\left(e^+e^- \to \text{hadrons}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

- Time scale for f^+f^- production: $\tau \sim 1/Q$
- Time scale for hadronization: $\tau \sim 1/\Lambda$

$$R \to 3\sum_q Q_q^2$$ (below Z-pole)

R



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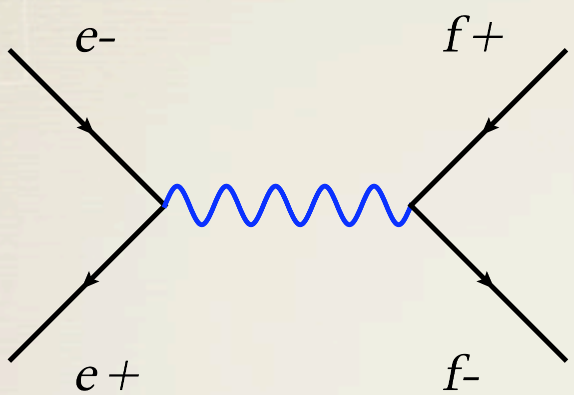

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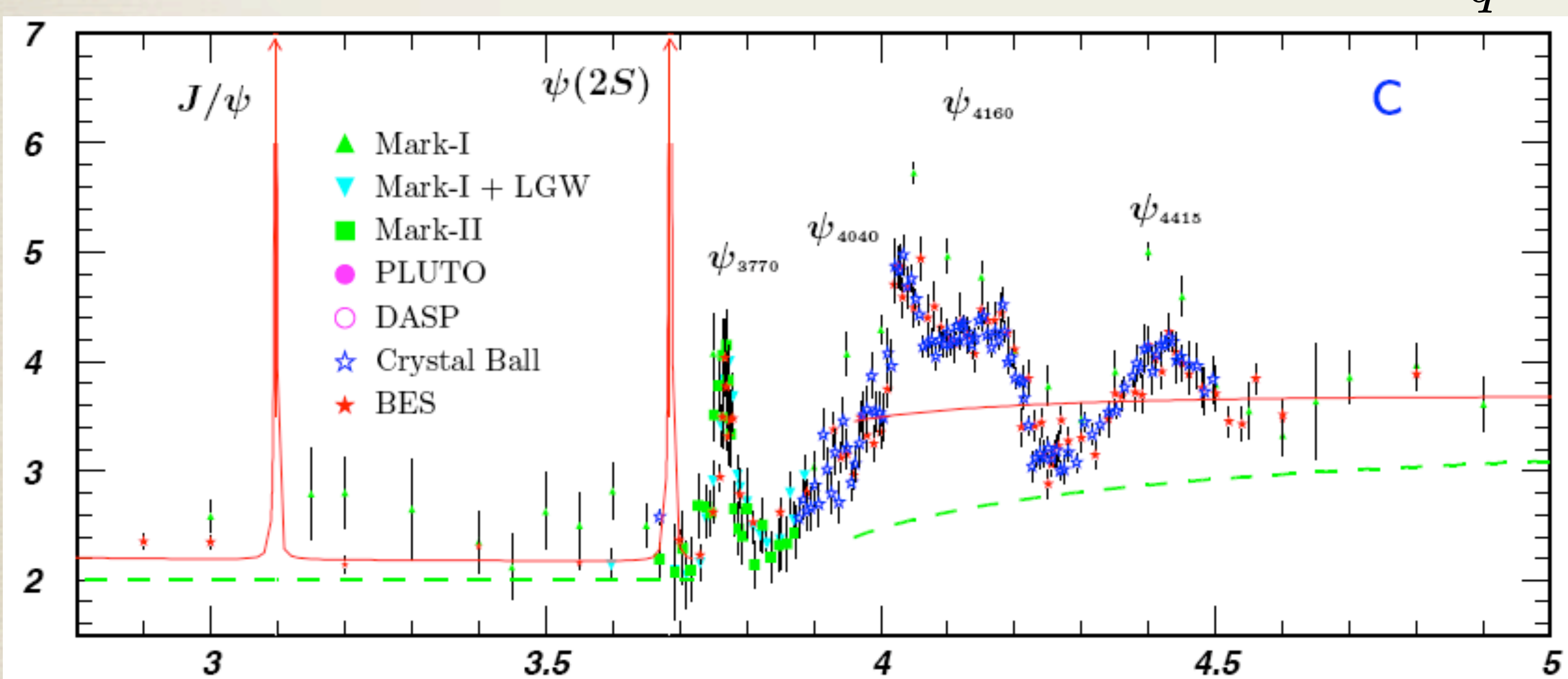
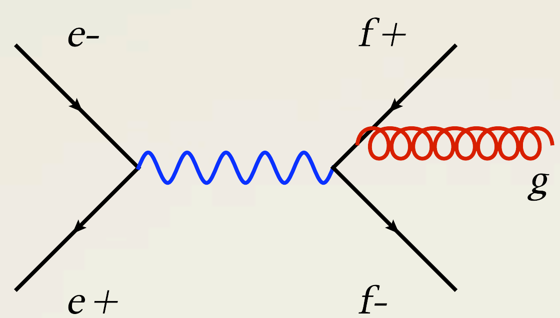
R

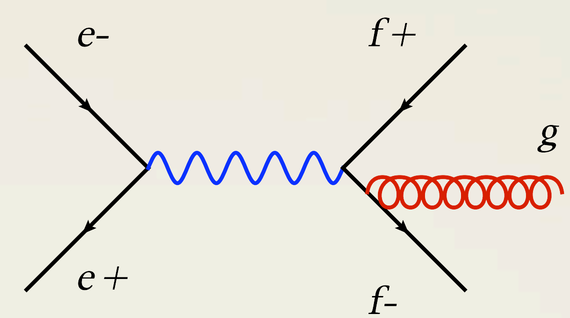


Goal: get the red line (or at least 1-loop pQCD)

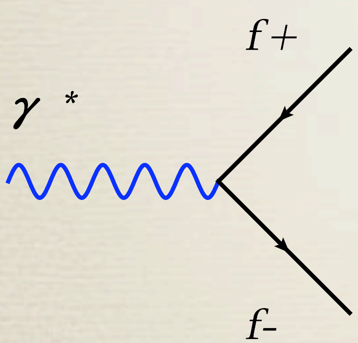
Real emission corrections

- What can happen in field theory? Can emit additional gluon.





- Work through this; since production part of $e^+e^- \rightarrow$ hadrons, $\mu^+\mu^-$ identical, can just consider $\gamma^* \rightarrow$ hadrons, $\mu^+\mu^-$ and form ratio



Leading-order matrix elements, phase space:

CM energy²

$$|\bar{\mathcal{M}}_0|^2 = \frac{1}{3}|\mathcal{M}_0|^2 = \frac{4e^2 Q_F^2 N_c\, s}{3}$$

3-d phase space

$$PS_0 = \frac{1}{2\sqrt{s}} \frac{1}{(2\pi)^2} \int d^d p_1 d^d p_2 \delta(p_1^2)\delta(p_2^2)\delta^{(d)}(p_\gamma - p_1 - p_2) = \frac{\Omega(3)}{64\pi^2\sqrt{s}}$$

$$R_0 = \frac{\sigma_{hadrons}}{\sigma_{\mu^+\mu^-}} = \frac{\left[|\bar{\mathcal{M}}_0|^2 \times PS_0\right]_{hadrons}}{\left[|\bar{\mathcal{M}}_0|^2 \times PS_0\right]_{\mu^+\mu^-}} = N_c \sum_q Q_q^2$$

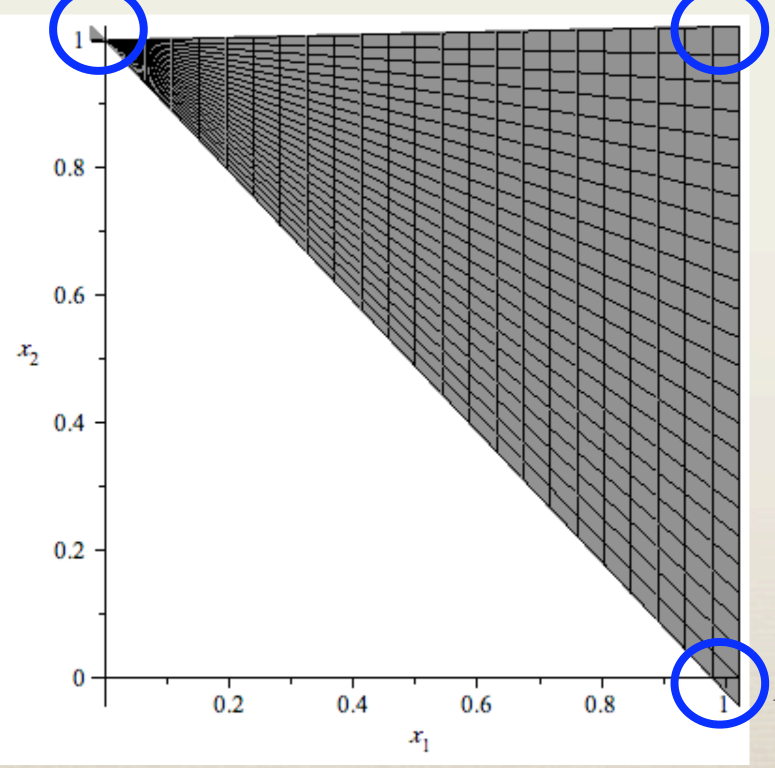
Real-emission phase space

$$PS_1 = \frac{1}{2\sqrt{s}}\frac{1}{(2\pi)^5}\int d^dp_1 d^dp_2 d^dp_g \delta(p_1^2)\delta(p_2^2)\delta(p_g^2)\delta^{(d)}(p_\gamma - p_1 - p_2 - p_g)$$

- Work in γ^* CM frame
- Introduce $x_1=2E_1/\sqrt{s}$, $x_2=2E_2/\sqrt{s}$

$$PS_1 = \sqrt{s}\,\frac{\Omega(2)\Omega(3)}{64(2\pi)^5}\int dx_1 dx_2 = PS_0 \times \frac{s}{16\pi^2}\int dx_1 dx_2$$

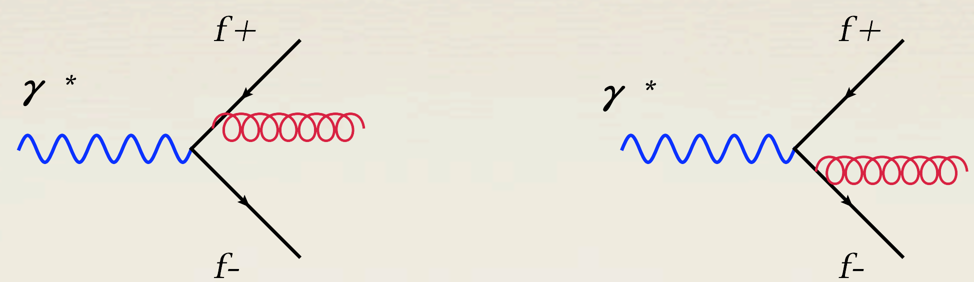
Quark carries no energy



Gluon carries no energy

Anti-quark carries no energy

Real-emission matrix elements



$s_{ij}=(p_i+p_j)^2$

$$
\begin{aligned}
|\bar{\mathcal{M}}_1|^2 &= 2C_F g_s^2 \frac{|\bar{\mathcal{M}}_0|^2}{s}\left\{\frac{s_{1g}}{s_{2g}}+\frac{s_{2g}}{s_{1g}}+2\frac{s\,s_{12}}{s_{1g}s_{2g}}\right\} \\
&= 2C_F g_s^2 \frac{|\bar{\mathcal{M}}_0|^2}{s}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}
\end{aligned}
$$

$$
R_1^{q\bar{q}g} = R_0 \times \frac{2g_s^2 C_F}{16\pi^2}\int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \,\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}
$$

⇨ **Singular** for $x_{1,2} \to 1$

$$
\begin{aligned}
s_{1g} &= 2E_1 E_g (1-\cos\theta_{1g}) \\
s_{2g} &= 2E_2 E_g (1-\cos\theta_{2g})
\end{aligned}
$$

collinear singularities for $p_g \parallel p_1$, $p_g \parallel p_2$

soft singularities when $E_g=(1-x_1-x_2)\sqrt{s}\to 0$

Dimensional regularization

- Taking d from 4 →4-2ε regulates both UV and IR divergences while maintaining gauge symmetries
- Coupling constant becomes dimensionful: $g_s^2 \to g_s^2 \mu^{2\varepsilon}$

$$PS_1 \to PS_0 \times \frac{s}{16\pi^2} \frac{1}{\Gamma(1-\epsilon)} \left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon} \int dx_1 dx_2 \left[\underbrace{(1-x_3)}_{x_3=2-x_1-x_2} (1-x_1)(1-x_2) \right]^{-\epsilon}$$

also recomputed in d-dimensions

For ε slightly negative, regulates $1/(1-x_{1,2})$

$$|\bar{\mathcal{M}}_1|^2 \quad \to \quad 2C_F g_s^2 \frac{|\bar{\mathcal{M}}_0|^2}{s} \left\{ \frac{(1-\epsilon)(x_1^2+x_2^2)+2\epsilon(1-x_3)}{(1-x_1)(1-x_2)} - 2\epsilon \right\}$$

$$\begin{aligned} R_1^{q\bar{q}g} \quad &= \quad R_0 \times \frac{2g_s^2 C_F}{16\pi^2\Gamma(1-\epsilon)} \left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon} \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \left\{ \frac{(1-\epsilon)(x_1^2+x_2^2)+2\epsilon(1-x_3)}{(1-x_1)(1-x_2)} - 2\epsilon \right\} \\ &\times \quad [(1-x_1)(1-x_2)(1-x_3)]^{-\epsilon} \end{aligned}$$

Infrared poles and KLN theorem

- Evaluate integrals to find:

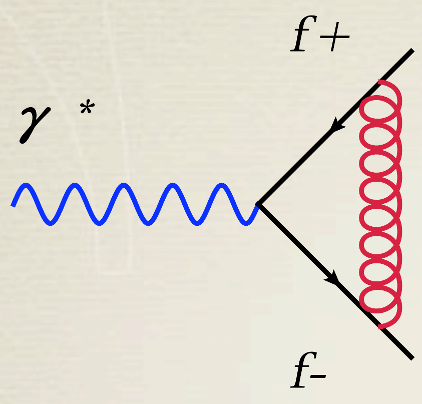
$$R_1^{q\bar{q}g} = R_0 \times \frac{\alpha_s C_F}{2\pi\Gamma(1-\epsilon)} \left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon} \left\{ \frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon) \right\}$$

double pole: soft+collinear gluon

single pole: soft or collinear gluon

- Regulator dependent! Not a physical observable.
- **KLN theorem**: singularities cancel if degenerate energy states summed over $\Rightarrow$ as gluon becomes soft or collinear, indistinguishable from virtual corrections, must add loops...

Virtual corrections

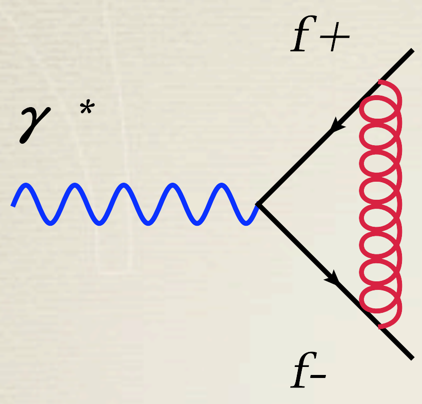


$$R_1^{q\bar{q}} = R_0 \times \frac{\alpha_s C_F \Gamma(1+\epsilon)}{2\pi} \left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon} \left\{-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon)\right\}$$

- ☑ As required by the KLN theorem, poles cancel upon addition of real and virtual corrections, leaving:

$$R = R_0 + R_1 + \mathcal{O}(\alpha_s^2) = R_0 \times \left\{1 + \frac{\alpha_s(\mu)}{\pi}\right\}$$

Virtual corrections



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- (A note about scaleless integrals: $\int d^d k \, \frac{1}{[k^2]^n} \propto \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} = 0$
- Very useful as long as you don't specifically care about the pole coefficients...)

Scale dependence

- Coupling constant depends on the parameter μ ⇒ dependence must vanish if we can compute to all orders

$$\frac{dR^{(n)}}{d\mu} \propto [\alpha_s(\mu)]^{n+1}$$

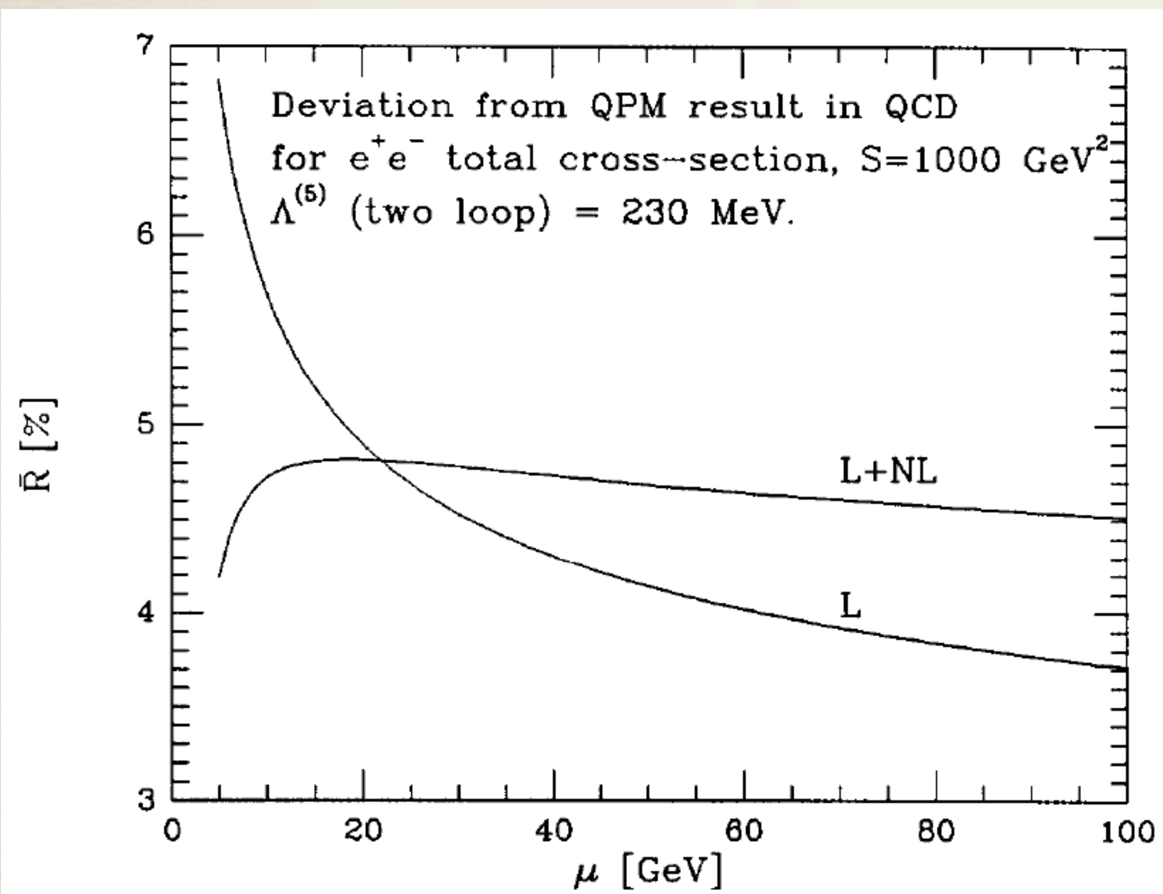
$$\begin{aligned} \frac{d\,\alpha_s(\mu)}{d\ln\mu} &= -\epsilon\alpha_s + \beta(\alpha_s) \\ \beta(\alpha_s) &= -2\,b_0\alpha_s^2 + \mathcal{O}(\alpha_s^3) \\ b_0 &= \frac{33 - 2N_F}{12\pi} \end{aligned}$$

- Can predict μ dependence of R_2

$$\begin{aligned} R^{(2)} &= R_0 + R_1 + R_2 + \mathcal{O}(\alpha_s^3) \\ \frac{d\,R^{(2)}}{d\ln\mu} &= \frac{d\,R_1}{d\ln\mu} + \frac{d\,R_2}{d\ln\mu} = -2\alpha_s^2\frac{b_0 R_0}{\pi} + \frac{d\,R_2}{d\ln\mu} \\ &\Rightarrow R_2 = \alpha_s^2\frac{b_0 R_0}{\pi}\ln\frac{\mu^2}{s} + (\mu\ \text{independent}) \end{aligned}$$

“Theoretical error”

- Variation of scale in some specified range is often used as an estimate of theoretical uncertainty ⇒ if it was calculated to higher orders, this dependence would vanish

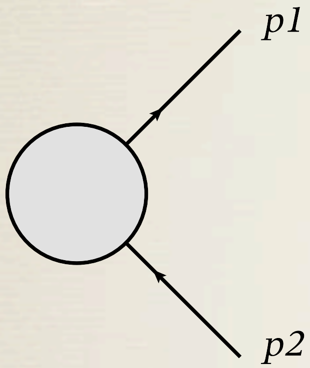


from Ellis, Stirling, Webber
QCD and Collider Physics

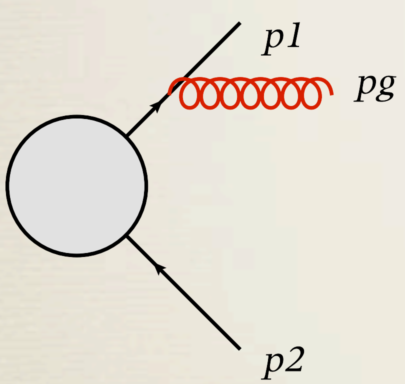
- Conventional range: $\sqrt{s}/2 \le \mu \le 2\sqrt{s}$
- Often underestimates LO→NLO, especially at hadron colliders where qualitatively new effects can appear at higher orders
- How to pick central value with multiple physical scales?

Eikonal approximation

- Useful to have diagnostic tools to check pieces of a calculation: ‘eikonal’ approximation for soft gluons gets double pole



$$= \bar{u}^i(p_1)\left[i\mathcal{M}_0^{ij}\right] v^j(p_2)$$



$$= \bar{u}^i(p_1)\left\{ i g_s \, \not{\epsilon}_g^a T^a_{ij} \frac{i(\not{p}_1 + \not{p}_g)}{(p_1+p_g)^2}\left[i\mathcal{M}_0^{jk}\right]\right\} v^k(p_2)$$

$$\approx -g_s \frac{p_1\cdot\epsilon_g^a}{p_1\cdot p_g}\bar{u}^i(p_1)\left\{T^a_{ij}\left[i\mathcal{M}_0^{jk}\right]\right\} v^k(p_2)$$

☑ Simplifies upon squaring matrix element; phase space also simplifies

soft region in gluon energy

$$d\sigma_S = \left[\frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_r^2}{s_{12}}\right)^{\epsilon}\right]\sum_{f,f'=1}^{4} d\sigma^0_{ff'}\int \frac{-p_f\cdot p_{f'}}{p_f\cdot p_5 \, p_{f'}\cdot p_5} dS$$

$$dS = \frac{1}{\pi}\left(\frac{4}{s_{12}}\right)^{-\epsilon}\int_0^{\delta_s\sqrt{s_{12}}/2} dE_5 E_5^{1-2\epsilon}\sin^{1-2\epsilon}\theta_1\, d\theta_1 \sin^{-2\epsilon}\theta_2\, d\theta_2$$

$$M^0_{ff'} = (\mathbf{T}^a_f \mathrm{M}_2)(\mathbf{T}^a_{f'}\mathrm{M}_2)^\dagger = \left[M_{c_1\ldots b_f\ldots b_{f'}\ldots c_4}\right]^* T^a_{b_f d_f} T^a_{b_{f'} d_{f'}} M_{c_1\ldots d_f\ldots d_{f'}\ldots c_4}$$

from Harris & Owens hep-ph/0102128, a useful reference for relevant formulae

color-connected Born amplitude; T^a operator has different expressions for quarks, gluons

Eikonal approximation

- Application to the current process yields:

$$R_{1,soft}^{q\bar{q}g} = R_0 \times \frac{\alpha_s C_F}{\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{s}{4\pi\mu^2}\right)^{-\epsilon} \left\{ \frac{1}{\epsilon^2} - \frac{2}{\epsilon} \ln\delta + 2\ln^2\delta + \text{finite} \right\}$$

agrees with our full calculation

Cutoff dependence must cancel against other regions of gluon phase space

- The 1/ε² poles must cancel against virtual corrections

Collinear approximation

- Another singular region to consider: collinear gluon emission. A simple way of calculating this phase-space region also exists. Study the region $p_1 \parallel p_g$.

  $s_{1g}=(1-x_2)s$
  $E_g=(1-x_1)\sqrt{s}/2$

  - already looked at soft gluons, restrict $x_1 \leq 1-\delta$
  - collinear region $s_{1g} \leq \delta_c s \Rightarrow x_2 \geq 1-\delta_c$

- Both matrix elements, phase space simplify.

only collinear emission

only hard gluons

$$R^{q\bar{q}g}_{1,1||g} \to R_0 \quad \times \quad \frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon}\int_{1-\delta_c}^{1} dx_2\,(1-x_2)^{-1-\epsilon}\int_0^{1-\delta} dx_1\,[x_1(1-x_1)]^{-\epsilon}$$

$$\times \quad \left\{\underbrace{C_F\frac{1+x_1^2}{1-x_1}}_{P_{qq}(x_1)}+\epsilon\underbrace{[-C_F(1-x_1)]}_{P'_{qq}(x_1)}\right\}$$

Unregulated splitting function

$$= \quad R_0 \times \frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\left[\frac{s}{4\pi\mu^2}\right]^{-\epsilon}\left\{\frac{1}{\epsilon}\left(\frac{3}{2}+2\ln\delta\right)-\ln^2\delta-\frac{3}{2}\ln\delta_c-2\ln\delta\ln\delta_c+\text{finite}\right\}$$

reproduces full result (with 2||g also);
cancels against virtual corrections

cancels against soft region (with 2||g also)

Phase-space slicing

- The splitting functions are universal, arbitrary matrix elements factorize in the collinear region.

$$\overline{\sum}|M_3(1+2\to 3+4+5)|^2 \simeq \overline{\sum}|M_2(1+2\to 3+4')|^2 P_{44'}(z,\epsilon) g^2 \mu_r^{2\epsilon}\frac{2}{s_{45}}$$

$$d\sigma_{\rm HC}^{1+2\to 3+4+5} = d\sigma_0^{1+2\to 3+4'}\left[\frac{\alpha_s}{2\pi}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu_r^2}{s_{12}}\right)^{\epsilon}\right]\left(-\frac{1}{\epsilon}\right)\delta_c^{-\epsilon}\int dz z^{-\epsilon}(1-z)^{-\epsilon}P_{44'}(z,\epsilon)$$

$$\begin{aligned}
P_{qq}(z) &= C_F\frac{1+z^2}{1-z}\\
P'_{qq}(z) &= -C_F(1-z)\\
P_{gq}(z) &= C_F\frac{1+(1-z)^2}{z}\\
P'_{gq}(z) &= -C_F z\\
P_{gg}(z) &= 2N\left[\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right]\\
P'_{gg}(z) &= 0\\
P_{qg}(z) &= \frac{1}{2}\left[z^2+(1-z)^2\right]\\
P'_{qg}(z) &= -z(1-z)\,,
\end{aligned}$$

- Forms the basis of an NLO subtraction scheme known as *phase-space slicing*
- Split full=soft+Σ(collinear)+hard; eikonal +collinear approximations to get singularities
- Numerical integration of hard region; dependence on ln(δ), ln(δ_c) must cancel

Parton Showers and Jets

Sudakov form factor

- Let's study again our real-emission cross section in the collinear limit, setting d=4.

$$d\sigma^{q\bar{q}g}_{collinear} \to \sigma_0 \frac{\alpha_s}{2\pi}\, dz\, P_{qq}(z) \sum_{t=s_{1g},s_{2g}} \frac{dt}{t}$$

$\Rightarrow$ independent emission of gluon from quark, anti-quark

- Focus on collinear region 1||g. Think of $1/\sigma_0 \times d\sigma^{qqg}$ as the probability of emitting gluon in interval dt. Also consider probability of no emission.

$$\begin{aligned} dP &= \frac{dt}{t}\frac{\alpha_s}{2\pi}\int dz\, P_{qq}(z) \\ dP_{no} &= 1 - \frac{dt}{t}\frac{\alpha_s}{2\pi}\int dz\, P_{qq}(z) \end{aligned}$$

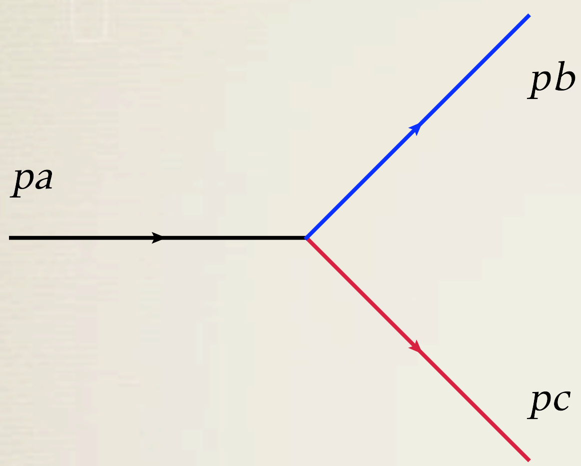
$\Rightarrow$ this *exponentiates*:

$$\Delta(t) = \exp\left\{-\int \frac{dt'}{t'}\frac{\alpha_s}{2\pi}\int dz\, P_{qq}(z)\right\}$$

Sudakov form factor, probability of no emission with invariant mass between upper, lower invariant masses.

Kinematics of the form factor

- Let's derive the limits of integration for Δ. Demand invariant masses be greater than some cutoff t_0



- Decompose momenta into *light-cone* components:

$$
\begin{aligned}
p^\mu &= \frac{n^\mu}{2} p^+ + \frac{\bar{n}^\mu}{2} p^- + p_T^\mu \\
n^\mu &= (1, \vec{0}, 1) \\
\bar{n}^\mu &= (1, \vec{0}, -1) \\
p^2 &= p^+ p^- - \vec{p_T}^{\,2}
\end{aligned}
$$

- Collinear emission of $p_{b,c}$:

$$
\begin{aligned}
p_b^\mu &= z p_a^+ \frac{n^\mu}{2} + p_T^\mu + p_b^- \frac{\bar{n}^\mu}{2} \\
p_c^\mu &= (1-z) p_a^+ \frac{n^\mu}{2} - p_T^\mu + p_c^- \frac{\bar{n}^\mu}{2}
\end{aligned}
$$

same as definition of z in example

smaller components

Constraints:

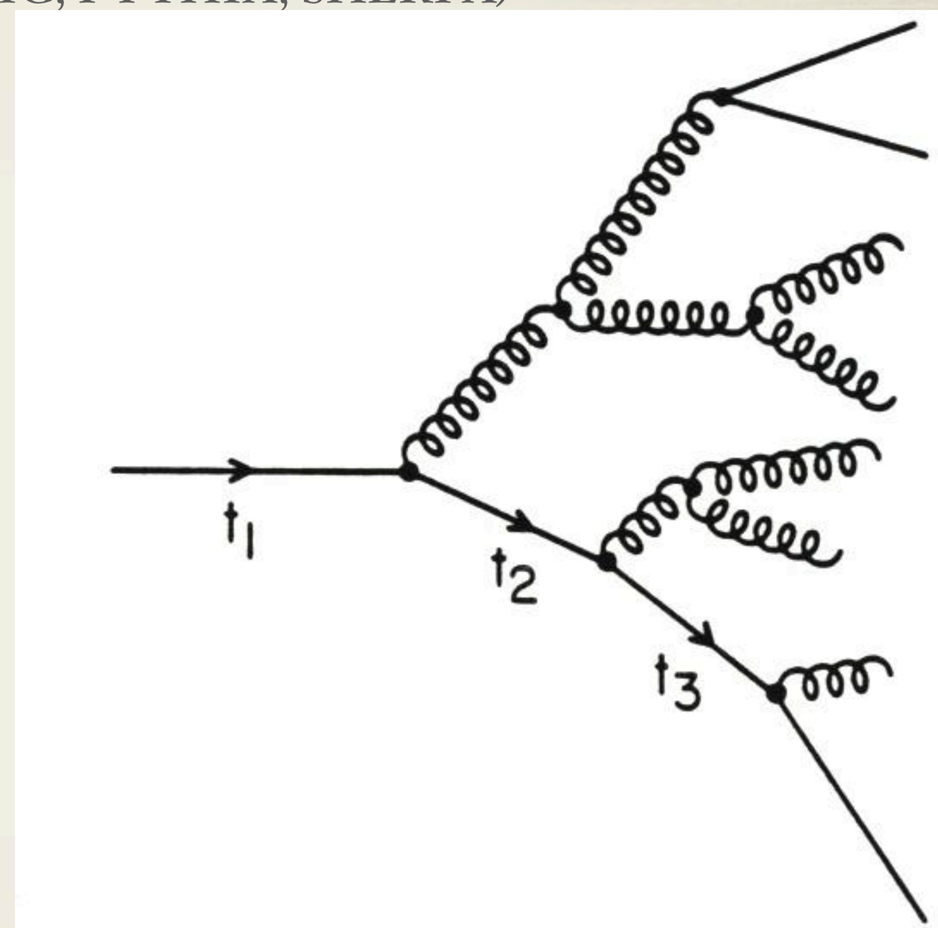
$$
\begin{aligned}
\vec{p_T}^{\,2} &= z(1-z)p_a^2 - (1-z)p_b^2 - z p_c^2 > 0 \\
z(1-z) &> \frac{t_0}{t}
\end{aligned}
$$

$$
\Delta(t) = \exp\left\{ -\int_{2t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_{\frac{t_0}{t'}}^{1-\frac{t_0}{t'}} dz\, P_{qq}(z) \right\}
$$

Parton shower Monte Carlo

- Can use to correctly (within collinear approximation) generate the emission of multiple partons (HERWIG, PYTHIA, SHERPA)

$$\Delta_i(t) = \exp\left\{-\sum_j \int \frac{dt'}{t'} \int dz \frac{\alpha_s}{2\pi} P_{ji}(z)\right\}$$



  - Generate r=Δ_2/Δ_1 ∋ [0,1], solve for t_2⇒evolves from t_1 to t_2 without emission
  - Generate energy fraction carried away by emission
  - Continue evolution with the two additional partons generated
  - Stop each evolution when r< Δ_1

- Not limited to invariant mass as an evolution variable; t=$E^2(1-\cos\theta)$, others possible

Leading logarithms

- Parton showers additionally resum *leading logarithms* that appear in the perturbative expansions of distributions

$$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

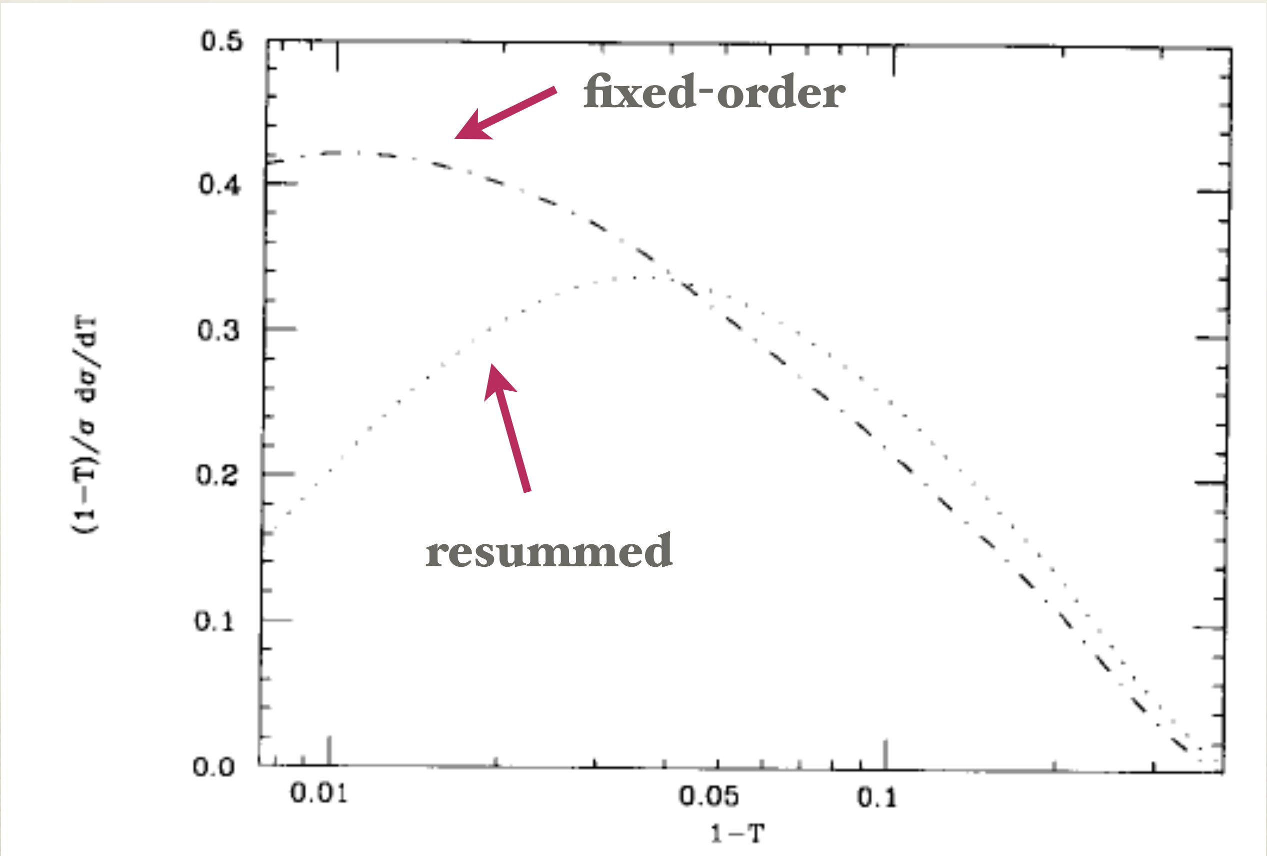
Near T=1:

$$\frac{1}{\sigma}\frac{d\sigma}{dT} \approx \frac{\alpha_s C_F}{\pi}\frac{1}{1-T}\ln\frac{1}{1-T}$$

Leading terms of the form $\alpha_S{}^n \ln^{2n-1}(1-T)/(1-T)$ resummed to all order in parton shower

$$R(\tau) = \int_\tau^1 dT \frac{1}{\sigma}\frac{d\sigma}{dT}$$

$$R(\tau) \sim \exp\left\{-\frac{\alpha_s C_F}{\pi}\ln^2(1-\tau)\right\}$$

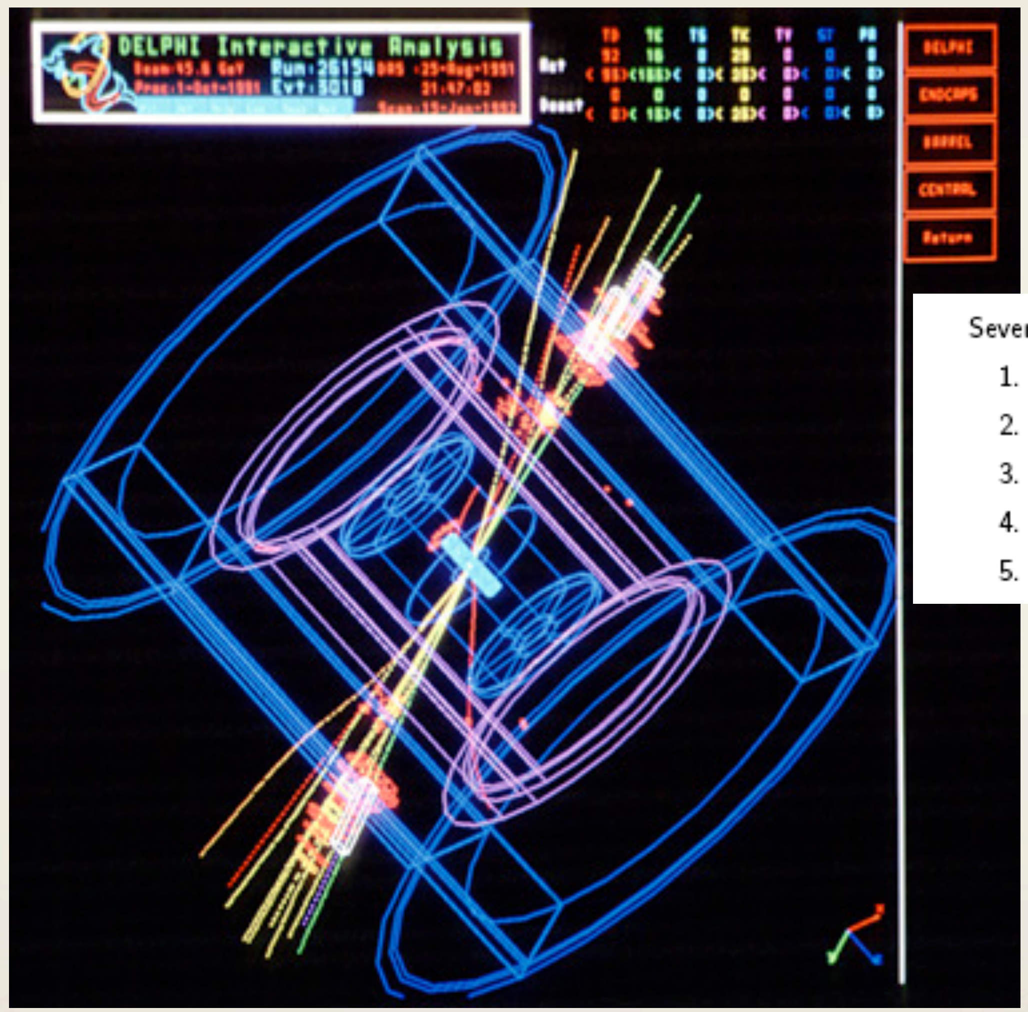


from Catani et al., NPB407 (1993)

Jets

- End-product partons fed into model of hadronization

- Should be a way to describe energy deposits without mention of specific hadrons: *jets*
- Specify a jet algorithm for combining the observed particles
- The idea: the jets should reflect the primordial hard partons



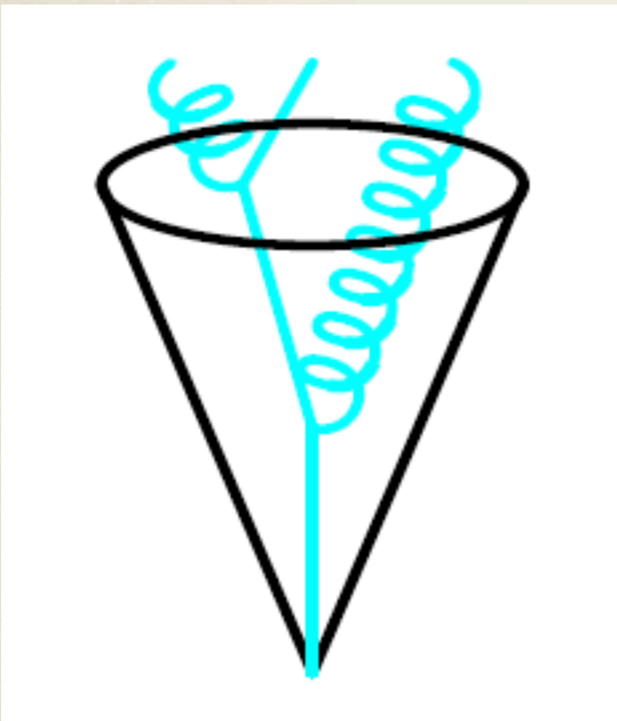
Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross sections at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronisation.

from G. Salam, 0906.1833, a useful review from which I will borrow

The cone

- Basic idea: draw a cone around the clusters of energy in the event



Iterated cones:

- Start with seed particle i (how to choose seeds?)
- Combine all particles within a cone of radius R

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 < R^2$$

rapidities   azimuthal angles

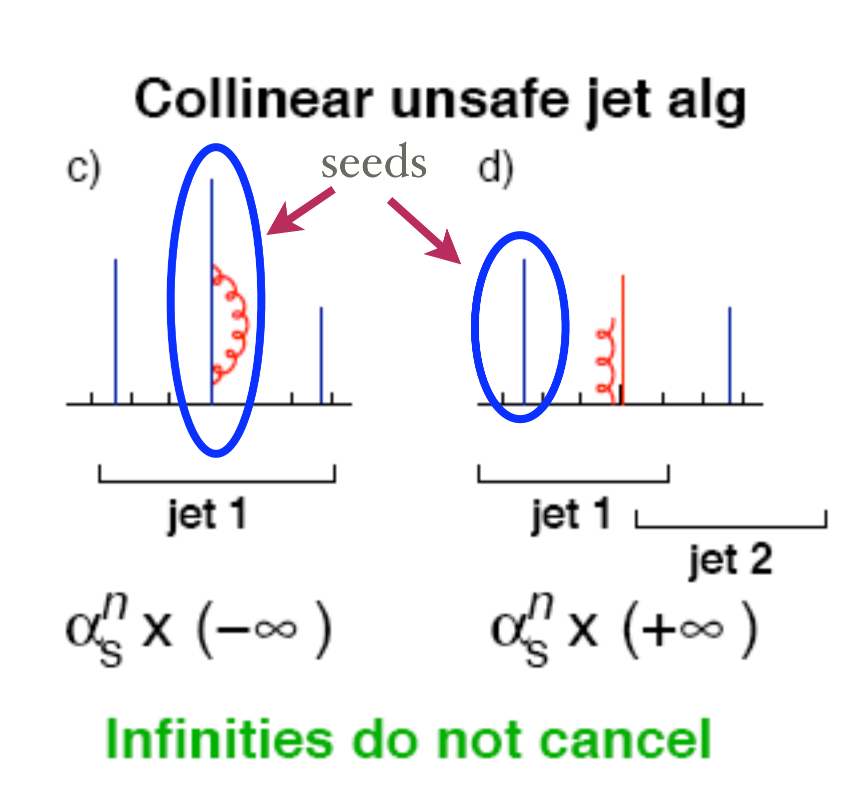
- Use the combined 4-momentum as a new seed
- Repeat until stability achieved

Examples:

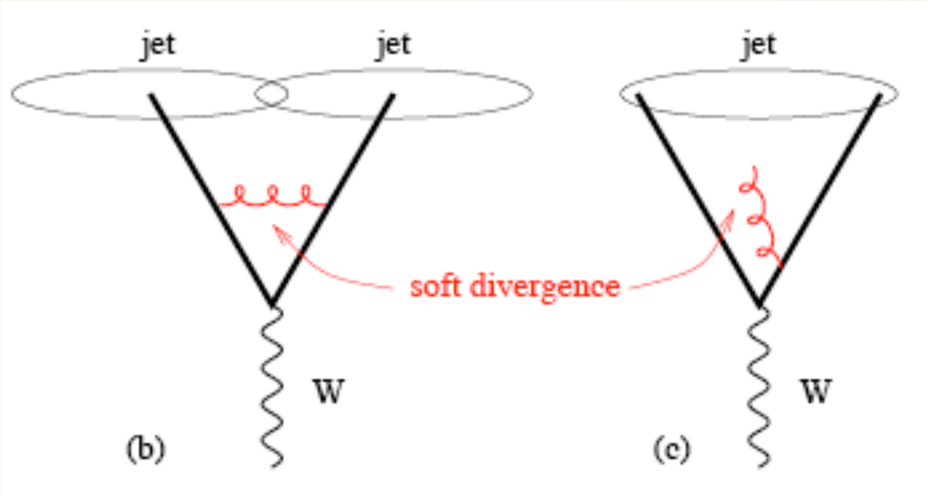
- Progressive removal (IC-PR): start with largest transverse momentum as seed; after finding stable cone, call it a jet and remove; go to next largest p_T
- Split-merge (IC-SM): first iterate all particles to find stable cones (``protojets"); consider p_T shared by protojets; if no shared p_T, call it a jet and remove; if $p_T^{\text{shared}}/p_T^{\text{2nd-hardest}} > f$, merge protojets; repeat

Infrared safety

- We saw before that IR singularities cancel between real, virtual corrections ⇒ *infrared safety*. Jet algorithm shouldn't spoil this cancellation. Both examples on previous slide do.



✗ IC-PR algorithm starts from different seed after emission of a hard collinear parton



✗ IC-SM algorithm contains new protojet after soft emission, that overlaps with other two; eventually merged

✗ ``Midpoint-cone" (include midpoint between stable cones as protojets) also IR unsafe

Consequences and the seedless cone

- Consequence: $1/\varepsilon \to \ln(p_T/\Lambda_{QCD}) \sim 1/\alpha_S \Rightarrow$ no suppression of higher-order contributions, no expansion possible

$$\underbrace{\alpha_s\alpha_{EW}}_{\text{LO}} + \underbrace{\alpha_s^2\alpha_{EW}}_{\text{NLO}} + \underbrace{\alpha_s^3\alpha_{EW}\ln\frac{p_t}{\Lambda}}_{\text{NNLO}} + \underbrace{\alpha_s^4\alpha_{EW}\ln^2\frac{p_t}{\Lambda}}_{\text{NNNLO}} + \cdots,$$

$$\sim \underbrace{\alpha_s\alpha_{EW}}_{\text{LO}} + \underbrace{\alpha_s^2\alpha_{EW}}_{\text{NLO}} + \underbrace{\alpha_s^2\alpha_{EW}}_{\text{NNLO}} + \underbrace{\alpha_s^2\alpha_{EW}}_{\text{NNNLO}} + \cdots$$

Observable	1st miss cones at	Last meaningful order
Inclusive jet cross section	NNLO	NLO
$W/Z/H + 1$ jet cross section	NNLO	NLO
3 jet cross section	NLO	LO
$W/Z/H + 2$ jet cross section	NLO	LO
jet masses in 3 jets, $W/Z/H + 2$ jets	LO	none

Situation for midpoint cone, from Salam & Soyez 0704.0292

- Can modify algorithms so that addition of soft particles doesn't modify hard jets in the event: SIScone (seedless infrared safe)

Salam & Soyez 0704.0292 and refs. within

Sequential recombination

- k_t algorithm:

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2)\frac{\Delta R_{ij}^2}{R^2}$$
$$d_{iB} = p_{ti}^2$$

  - Work out all d_{ij}, d_{iB}, find minimum
  - If it is a d_{ij}, combine i and j and restart
  - If it is a d_{iB}, call i a jet and remove it
  - Stop after no particles remain

- Generalizations use a slightly different distance measure

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p})\frac{\Delta R_{ij}^2}{R^2}$$
$$d_{iB} = p_{ti}^{2p}$$

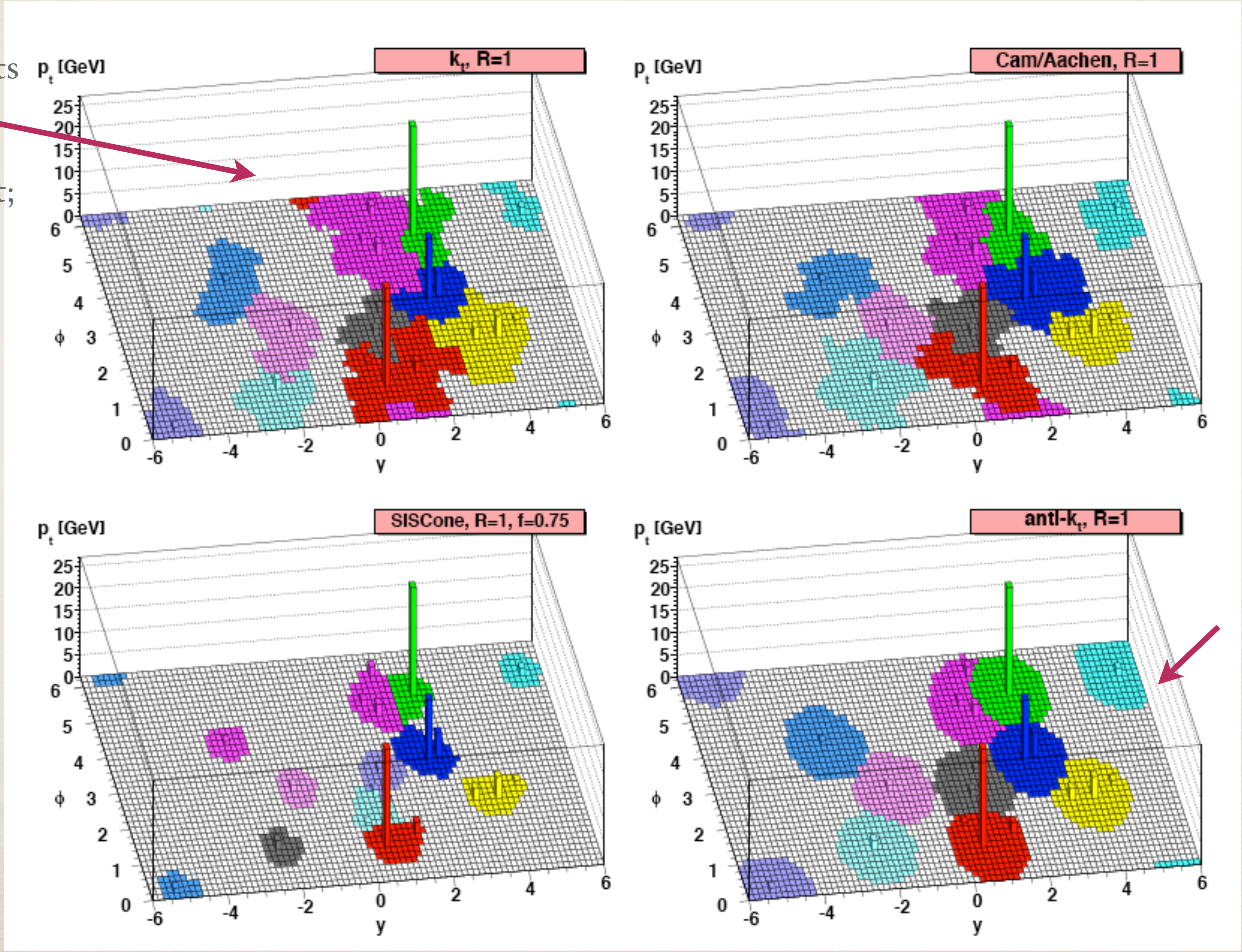
  - p=-1: anti-k_t
  - p=0: Cambridge-Aachen

- Roughly, soft and collinear emissions come with small distance measure and are always recombined $\Rightarrow$ IR safe

Jets in pictures

- Areas denote where soft radiation would be "soaked up" by jet

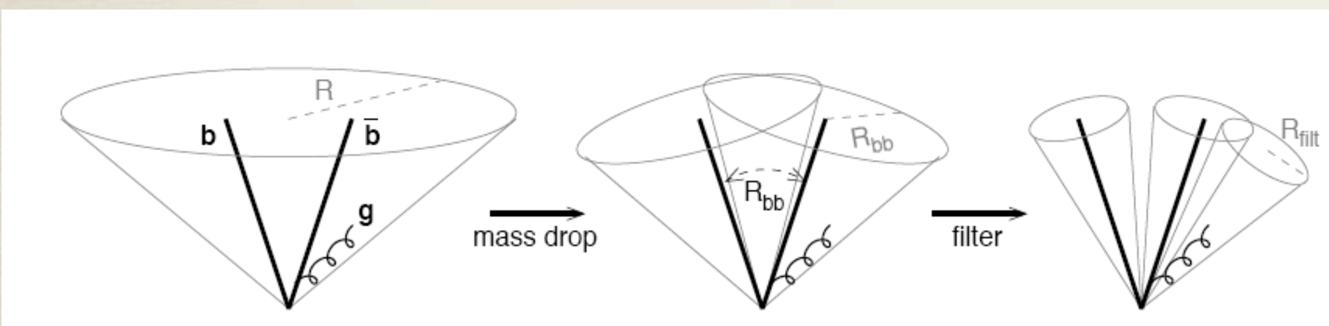
First clusters all sorts of soft particles, which eventually become added to jet; more sensitive to underlying event, pile-up



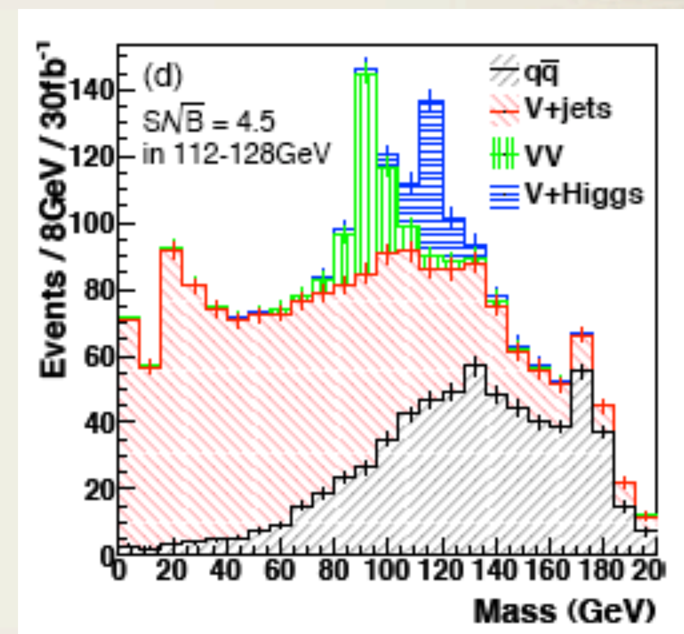
Avoids this with the $1/p_t^2$ in d_{ij}

Jet substructure

- Recent interest in using substructure of jets to distinguish signal from background. For example, highly-boosted Higgs will produce a "fat jet" with two b subjets inside.



Undo last stage of clustering and look for significant mass drop, consistent with heavy particle decaying to jets



Butterworth et al., 0802.2470

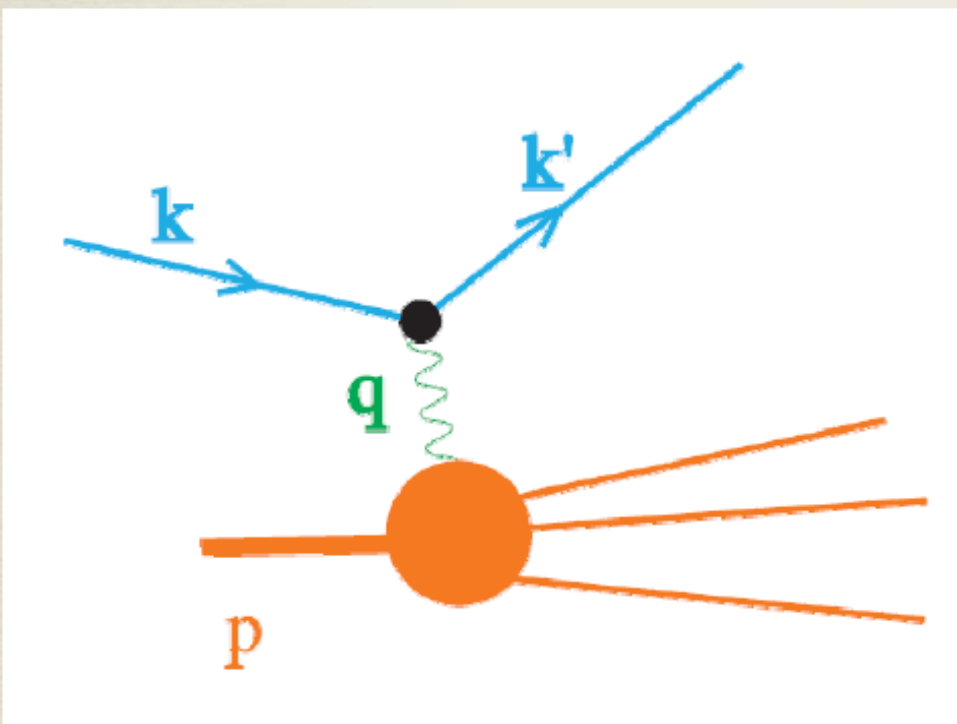
- Boosted tops, W/Z bosons have been studied in various contexts

G. Salam, 0906.1833 for refs

Example 2: Deep inelastic scattering and PDFs

Deep inelastic scattering

- Putting one hadron in the initial state leads to DIS $\Rightarrow$ still gives most of our information on PDFs (ep at DESY)



Kinematics:

$$\begin{aligned} q^\mu &= k^\mu - k'^\mu \\ Q^2 &= -q^2 \\ x &= \frac{Q^2}{2P\cdot q} \\ y &= \frac{P\cdot q}{P\cdot k} \overset{lab}{=} \frac{E-E'}{E} \end{aligned}$$

$$d\sigma = \frac{4\alpha^2}{s}\frac{d^3\vec{k}'}{2|\vec{k}'|}\frac{1}{Q^4}L^{\mu\nu}(k,q)W_{\mu\nu}(p,q)$$

phase space scat. lepton

photon propagator²

leptonic tensor

hadronic tensor contains information about hadronic structure

Hadronic tensor

- Hermiticity, parity, current conservation allow us to simplify $W_{\mu\nu}$

EM current

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\langle P|J^\dagger_\nu(z) J_\mu(0)|P\rangle \\ &= \left\{g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right\} F_1(x,Q^2) + \left\{P_\mu + \frac{q_\mu}{2x}\right\}\left\{P_\nu + \frac{q_\nu}{2x}\right\}\frac{F_2(x,Q^2)}{P\cdot q} \end{aligned}$$

Structure functions

$$\frac{d\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left\{\left[1+(1-y)^2\right]\,F_1 + \frac{1-y}{x}\left[F_2 - 2x\,F_1\right]\right\}$$

- *Factorization* tells us that EM probe scatters off partons

PDFs

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\int_0^1 \frac{d\xi}{\xi}\sum_a f_a(\xi)\langle p|J^\dagger_\nu(z)J_\mu(0)|p\rangle_{p=\xi P}$$

(Note: this really is field theory: $f_q(\xi) = \int \frac{dx^-}{4\pi}\langle P|\bar{q}_a(y^-)\gamma^+ W(y^-,0)_a(0)|P\rangle$)

Wilson line

Calculating the structure function

- Calculate by inserting a complete set of states between currents; at LO, have a single-quark final state:

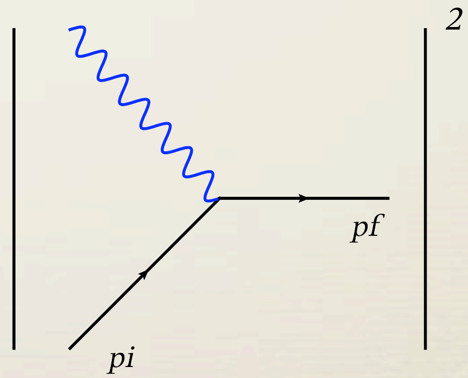
$$\begin{aligned} P^\mu &= \frac{Q}{2x}\left(1,\vec{0},1\right) \\ p^\mu &= \frac{\xi Q}{2x}\left(1,\vec{0},1\right) \\ q^\mu &= \left(0,\vec{0},-Q\right) \end{aligned}$$

$$\begin{aligned} PS &= \int \frac{d^d p_f}{(2\pi)^{d-1}} \delta(p_f^2)(2\pi)^d \delta^{(d)}(q+p-p_f) \\ &= \frac{2\pi}{Q^2}\delta\left(1-\frac{x}{\xi}\right) \end{aligned}$$

- Can isolate F_2 with a projection operator:

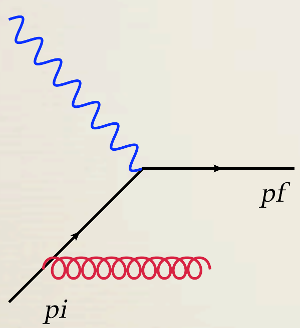
$$\begin{aligned} F_2 &= R^{\mu\nu} W_{\mu\nu} \\ R^{\mu\nu} &= \frac{2x}{d-2}\left\{ g^{\mu\nu} - 4\,(d-1)\frac{x^2}{Q^2} P^\mu P^\nu \right\} \end{aligned}$$

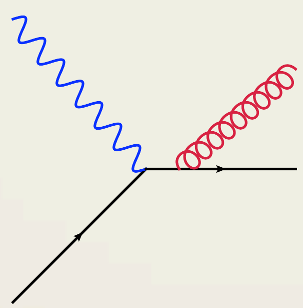
$$\begin{aligned} F_2 = \frac{1}{4\pi}\int \frac{d\xi}{\xi} \sum_q f_q(\xi) \times \frac{PS}{2N} \times R^{\mu\nu} \times & \\ = \sum_q e^2 Q_q^2 \int d\xi\, f_q(\xi)\, \xi\, \delta(x-\xi) & \\ = \sum_q e^2 Q_q^2\, x\, f_q(x) & \end{aligned}$$

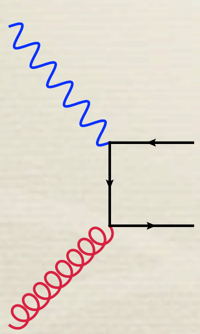


Scaling

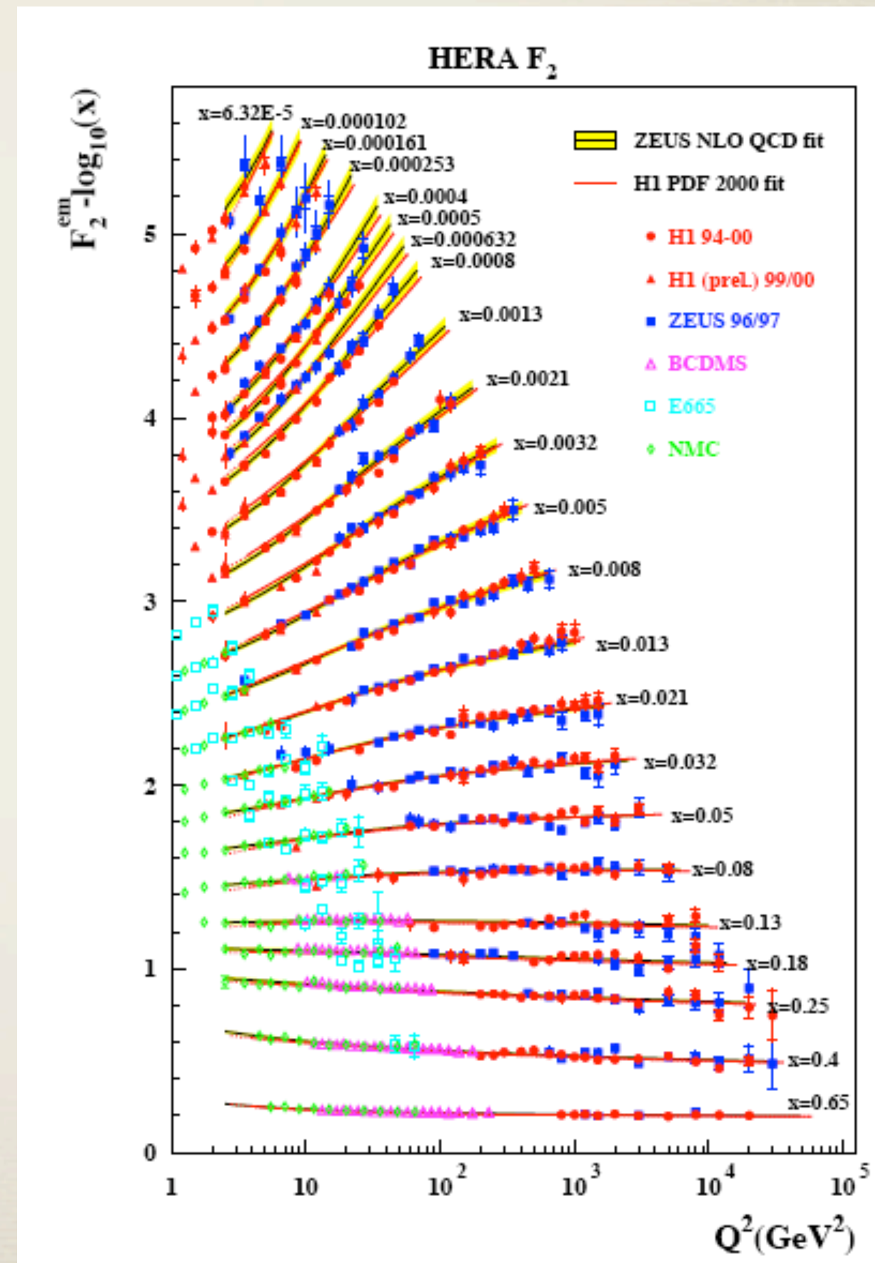
- No Q^2 dependence in $F_2 \Rightarrow$ ***scaling***, comes from scattering off point-like constituents of proton
  - Clearly a good approximation, but also clearly violated
  - Goal: check to see that QCD reproduces the scaling violation
  - Possible NLO real-emission terms:







$\Rightarrow$ we'll do the quark pieces and quote the answer for these



Real-emission phase space

- Focus on new aspects with respect to $e^+e^- \to$ hadrons; first, derive a useful parameterization of the phase space

$$
\begin{aligned}
PS &= \frac{1}{(2\pi)^{d-2}} \int d^d p_f d^d p_g \delta(p_g^2)\delta(p_f^2) \\
&\times \delta^{(d)}(q+p-p_f-p_g) \\
&= \frac{\Omega(d-2)}{4(2\pi)^{d-2}} \int_0^1 \left[Q^2 y(1-y)\frac{\xi}{x}\left(1-\frac{x}{\xi}\right)\right]^{-\epsilon} \\
p \cdot p_g &= \frac{\xi}{2x} Q^2 y \\
p_f \cdot p_g &= \frac{\xi}{2x} Q^2 \left(1-\frac{x}{\xi}\right)
\end{aligned}
$$

Real-emission matrix elements

- Spin, color summed/averaged+projected matrix elements; focus on the potentially divergent terms

$$|\bar{\mathcal{M}}|^2 = 4\,C_F\,e^2 Q_q^2\,g_s^2 \mu^{2\epsilon} \left\{ \frac{p_f \cdot p_g}{p \cdot p_g} + \frac{p \cdot p_g}{p_f \cdot p_g} + \frac{Q^2 p \cdot p_f}{p_f \cdot p_g\, p \cdot p_g} + \underbrace{\ldots}_{\text{finite terms}} \right\}$$

- Need to integrate over y, include $\frac{1}{4\pi}\int \frac{d\xi}{\xi} f_q(\xi)$

$$F_{2,q}^{(1),real} = e^2 Q_q^2\, x\, \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left[\frac{Q^2}{4\pi\mu^2}\right]^{-\epsilon} \left(\frac{x}{\xi}\right)^{\epsilon} \left(1-\frac{x}{\xi}\right)^{-\epsilon}$$

$$\times \int_x^1 \frac{d\xi}{\xi} f_q(\xi) \left\{ -\frac{C_F}{\epsilon} \frac{1+(x/\xi)^2}{1-x/\xi} - 2C_F \frac{x/\xi}{1-x/\xi} + \ldots \right\}$$

This term is bad news, no way it can cancel against virtual correction, which are δ(x-ξ)

Looks like P_{qq} ⇒ collinear singularity

Notice the singularity when x= ξ ⇒ soft singularity

Factorization of IR singularities

- Solution: must absorb initial-state collinear singularity into PDF. Redo calculation with $f_q \rightarrow f_{q,0}$, a bare PDF. Choose the bare PDF to remove $1/\varepsilon$ pole. PDF sensitive to IR QCD.
- Must also add virtual corrections, deal with the x=ξ soft singularity of real emission. Correct way to do so is with *plus distributions*.

$$\int_0^1 dx\, f(x)\,[g(x)]_+ = \int_0^1 dx\, g(x)\,[f(x) - f(0)]$$ $\Rightarrow$ if g ~1/x, removes singularity

$$F_{2,q} = e^2 Q_q^2\, x \int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) \left\{ \delta(1 - x/\xi) + \frac{\alpha_s}{2\pi\Gamma(1-\epsilon)} \left[\frac{Q^2}{4\pi\mu^2}\right]^{-\epsilon} \left[-\frac{1}{\epsilon} P_{qq}(x/\xi) + \text{finite}\right]\right\}$$

$$P_{qq}(x) = C_F \left[\frac{1+x^2}{[1-x]_+} + \frac{3}{2}\delta(1-x)\right] \left(\Rightarrow \int_0^1 P_{qq}(x) = 0\right)$$ ← quark-number conservation

$$f_q(x,\mu^2) = f_{q,0}(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) \left\{-\frac{1}{\epsilon} P_{qq}(x/\xi) + C(x/\xi)\right\}$$ ← $\overline{\text{MS}}$: C chosen to remove $\ln(4\pi)-\gamma_E$

$$F_{2,q} = e^2 Q_q^2\, x \int_x^1 \frac{d\xi}{\xi} f_q(\xi,\mu^2) \left\{ \delta(1 - x/\xi) + \frac{\alpha_s}{2\pi} \left[P_{qq}(x/\xi) \ln\frac{Q^2}{\mu^2} + \text{finite}\right]\right\}$$

Scale variation and DGLAP

- Pole turns into a $\ln(\mu^2)$ dependence $\Rightarrow$ F_2 must be independent of this arbitrary *factorization scale*, which leads to an evolution equation for the PDF.

$$\frac{d\, f_q(x,\mu^2)}{d\ln\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} f_q(\xi,\mu^2) P_{qq}(x/\xi) \qquad \Rightarrow \textbf{\textit{DGLAP equation}}$$

☑ Leads to a $\ln(Q^2)$ dependence of F_2 $\Rightarrow$ explains the observed scaling violation

- Inclusion of the gluon-initiated partonic processes:

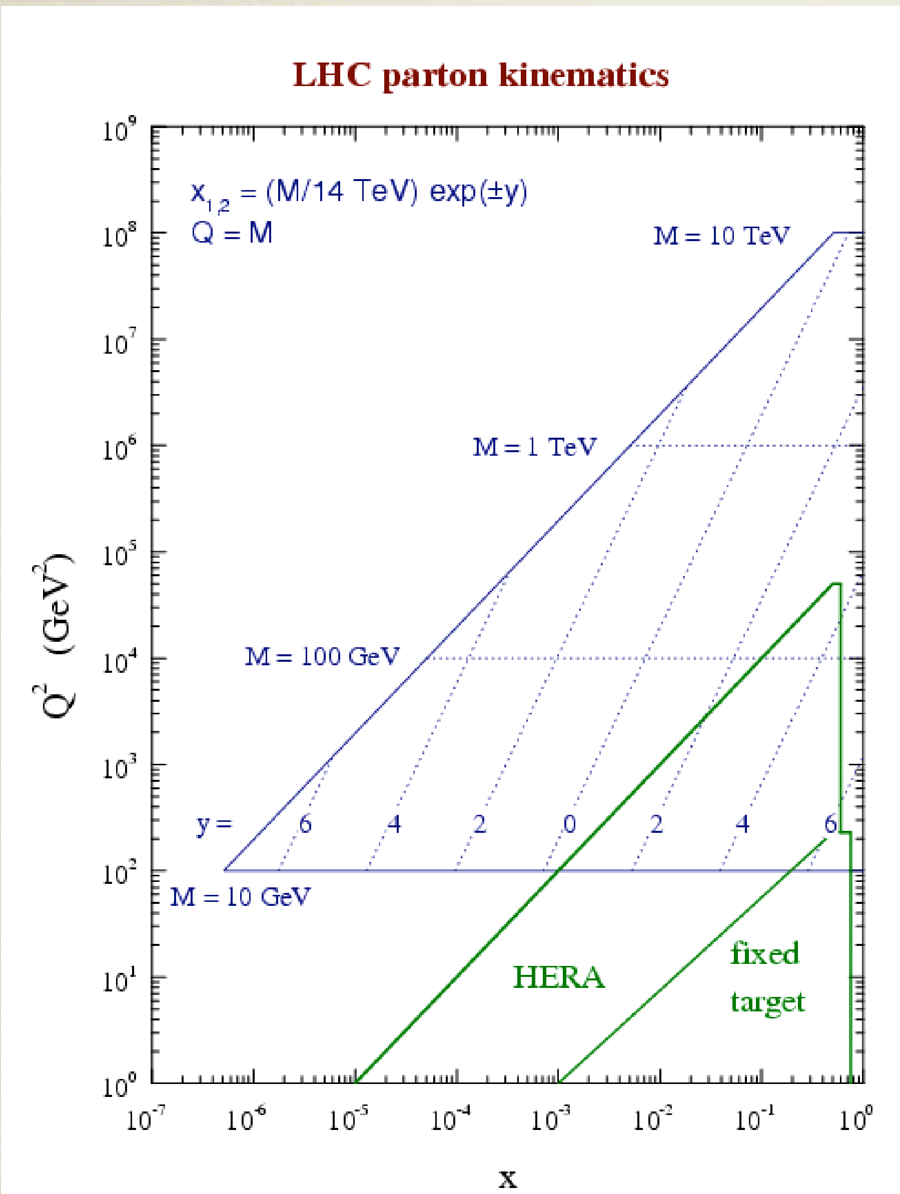
$$\begin{aligned} F_{2,q} &= e^2 Q_q^2\, x \int_x^1 \frac{d\xi}{\xi} f_q(\xi,\mu^2) \left\{ \delta(1-x/\xi) + \frac{\alpha_s}{2\pi}\left[P_{qq}(x/\xi)\ln\frac{Q^2}{\mu^2} + \text{finite}\right]\right\} \\ &+ e^2 Q_q^2\, x \int_x^1 \frac{d\xi}{\xi} f_g(\xi,\mu^2) \left\{ \frac{\alpha_s}{2\pi}\left[P_{qg}(x/\xi)\ln\frac{Q^2}{\mu^2} + \text{finite}\right]\right\} \end{aligned}$$

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} f_q(x,\mu^2) \\ f_g(x,\mu^2)\end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\begin{pmatrix} P_{qq}(x/\xi) & P_{qg}(x/\xi) \\ P_{gq}(x/\xi) & P_{gg}(x/\xi)\end{pmatrix}\begin{pmatrix} f_q(x,\mu^2) \\ f_g(x,\mu^2)\end{pmatrix}$$

Determining PDFs

- Enter every hadron collider prediction!

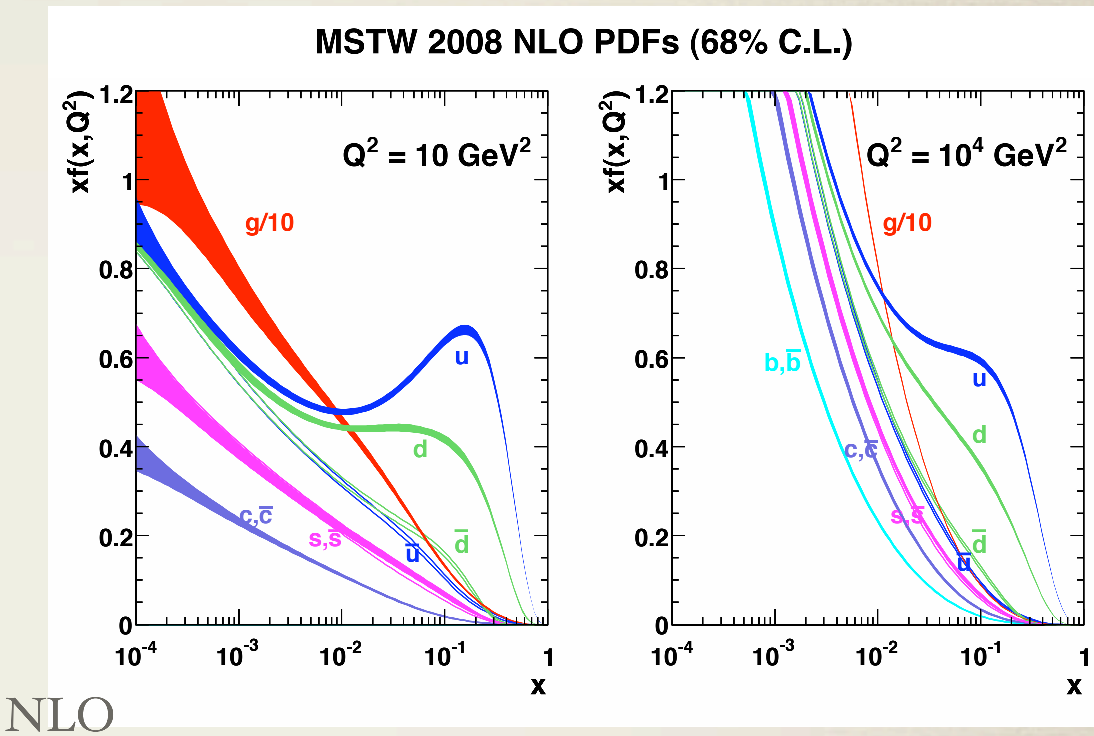
Lots of gluons!



Fits by CTEQ, MSTW, ABKM, NNPDF

DIS, fixed-target DY, Tevatron jets +W,Z

Only known at NLO

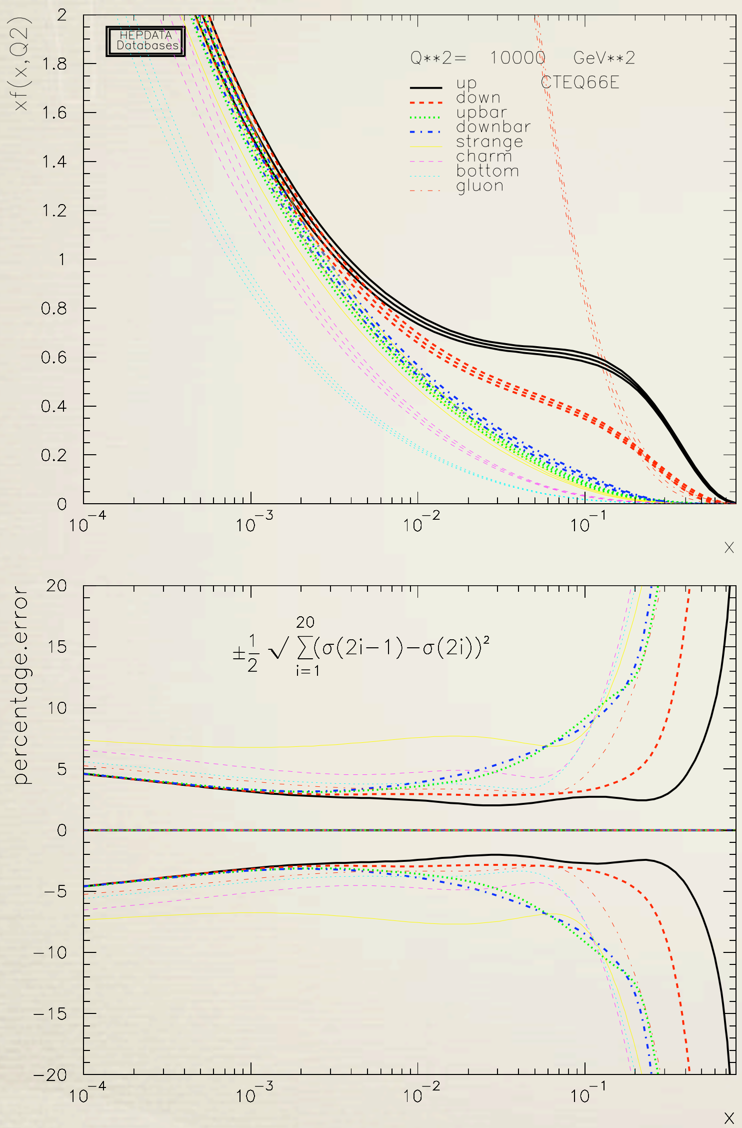


Process	Subprocess	Partons	x range
$\ell^\pm\{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu})N \to \mu^-(\mu^+)X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu N \to \mu^-\mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \to \bar{\nu} X$	$W^+\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
$e^\pm p \to e^\pm c\bar{c} X$	$\gamma^* c \to c,\ \gamma^* g \to c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm\nu)X$	$ud \to W, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$p\bar{p} \to (Z \to \ell^+\ell^-)X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$

Fixed target HERA TeV

PDF errors

- Published sets come with errors... what do they mean?



CTEQ 6.6, http://durpdg.dur.ac.uk/

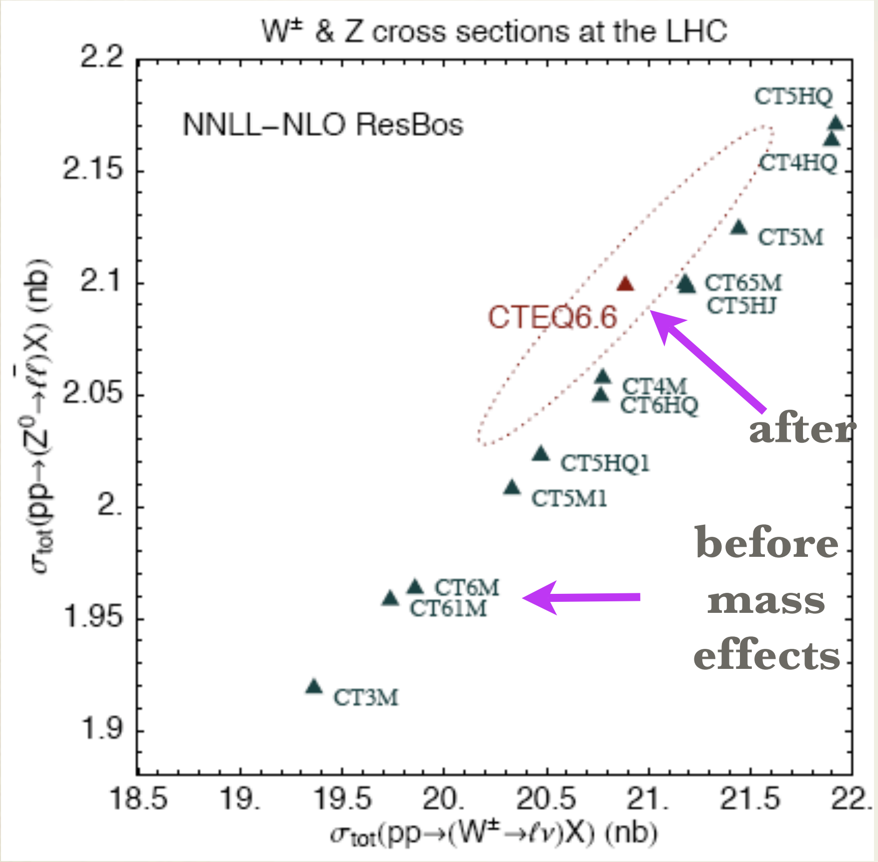
- There are many sources of uncertainty in the PDFs, some of which we've touched on
  - Data set choice
  - Kinematic cuts
  - Parametrization choices
  - Treatment of heavy quarks, target mass corrections, and higher twist terms
  - Order of perturbation theory
  - Errors on the data ➡ ***Only* error included!**
- Techniques have been developed to handle the last one
- The others require judgement and experience, but *are not* included in what are generally referred to as PDF errors.

Review by J. Owens at CTEQ 2007 summer school, http://www.phys.psu.edu/~cteq/schools/summer07/

Two recent examples...

PDF error examples

CTEQ, P. Nadolsky et al. '08



MSTW 2008 PDF release arXiv:0901.0002

- Run II inclusive jet data
- Quark-mass effects
- Gluon density decreased at x~0.1

M_H=170 GeV Higgs at Tevatron (pb):

MRST 2001	MRST 2004	MRST 2006	MSTW 2008
0.3833	0.3988	0.3943	0.3444

Anastasiou, Boughezal, FP 0811.3458

Inclusion of m_c, m_b suppresses F_2 at low $Q^2 \Rightarrow$ increase u,d to compensate

6-7% increase in LHC W, Z predictions

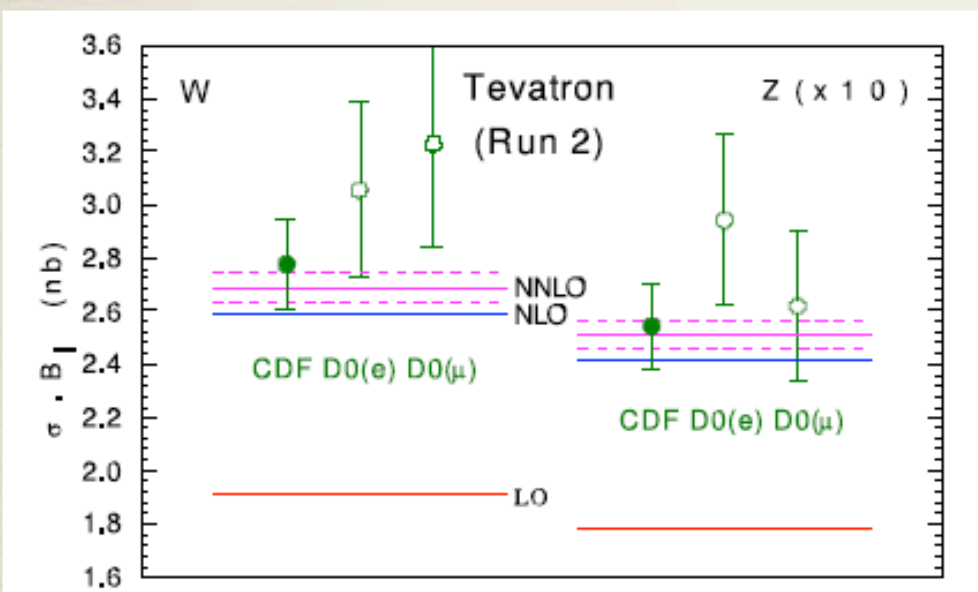
~10-15% decrease in predicted cross section !
Previous 90% CL error: ±5%

Keep in mind for LHC applications...

Example 3: Higgs production at NLO

NLO at hadron colliders

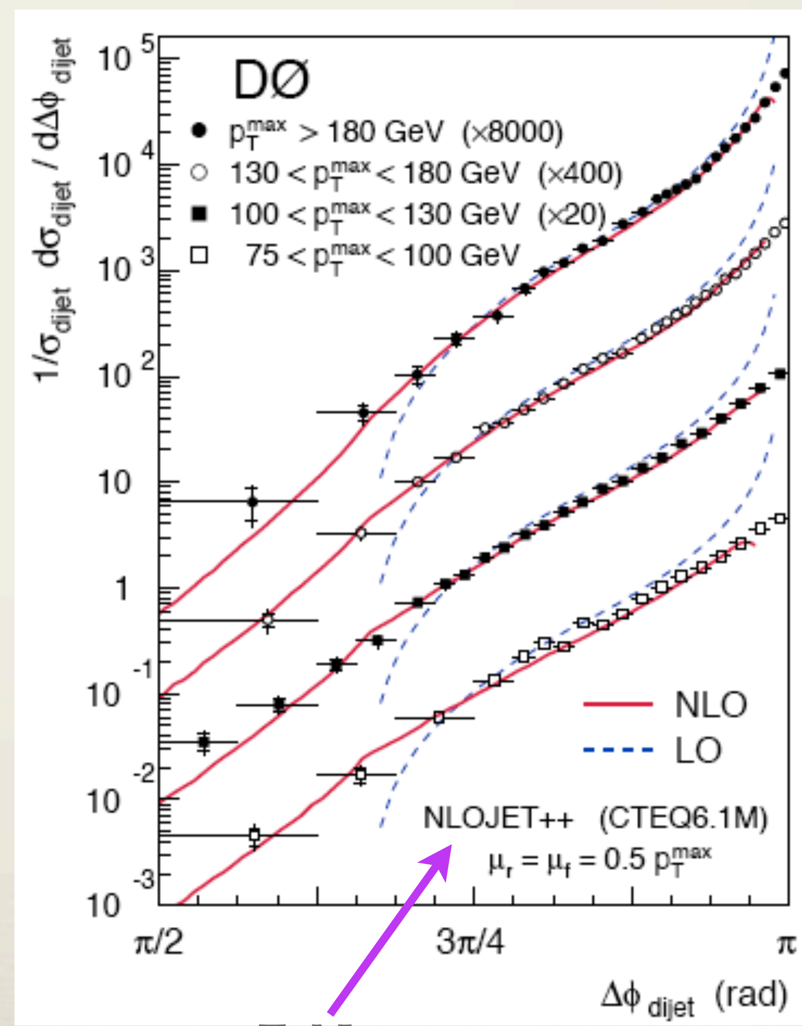
- Improved normalization and smaller residual uncertainty
- Better description of distribution shapes
- First serious quantitative prediction only at NLO

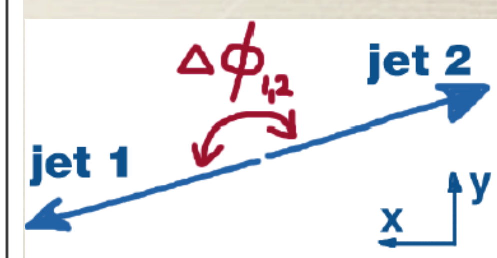


W+jets

number of jets	CDF	LO	NLO
1	53.5 ± 5.6	$41.40(0.02)^{+7.59}_{-5.94}$	$57.83(0.12)^{+4.36}_{-4.00}$
2	6.8 ± 1.1	$6.159(0.004)^{+2.41}_{-1.58}$	$7.62(0.04)^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.796(0.001)^{+0.488}_{-0.276}$	$0.882(0.005)^{+0.057}_{-0.138}$

BLACKHAT: Berger et al., 0907.1984



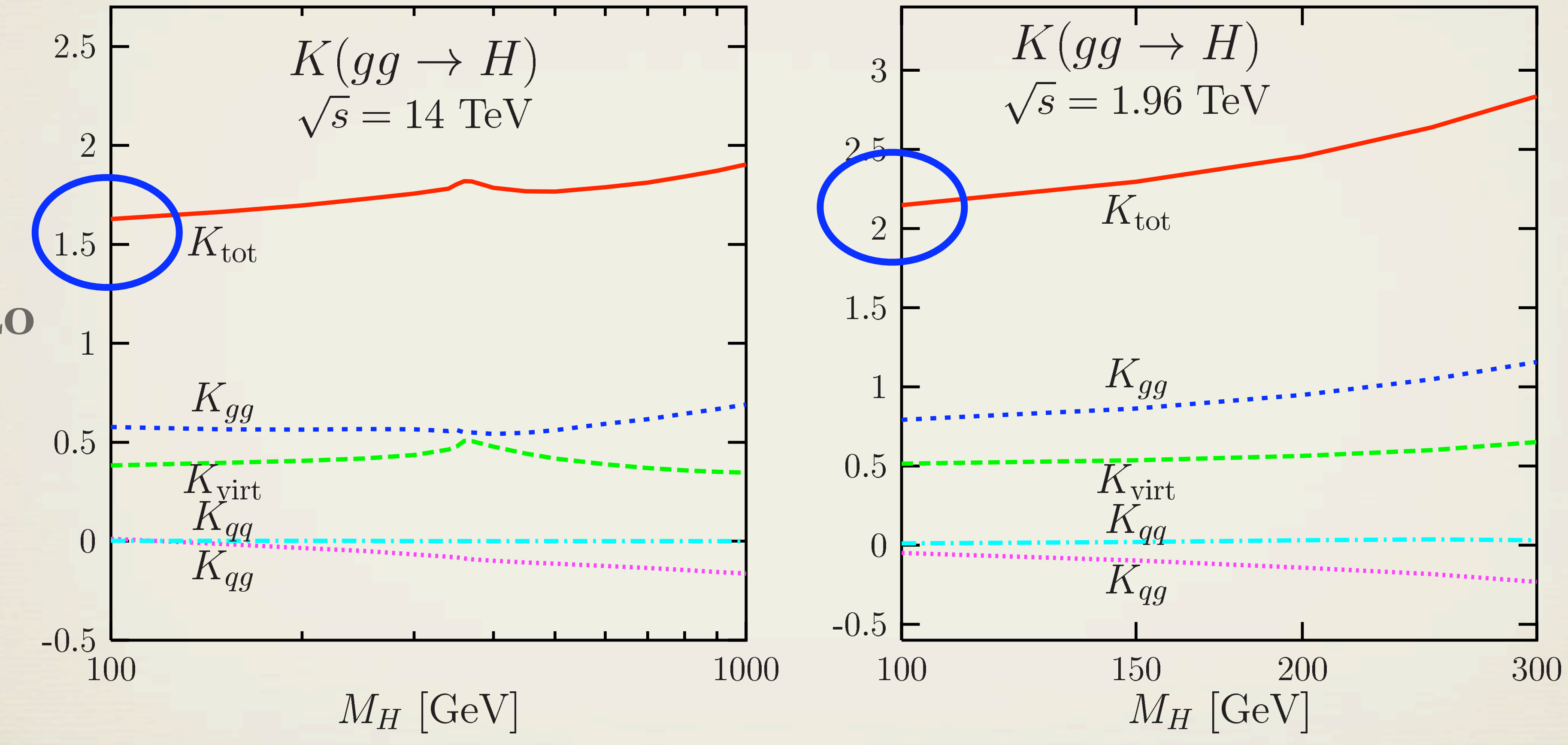


Z. Nagy

Higgs production in gluon fusion

- Naive estimate of magnitude: α_s/π~few percent; why so large?

$K=\sigma_{NLO}/\sigma_{LO}$



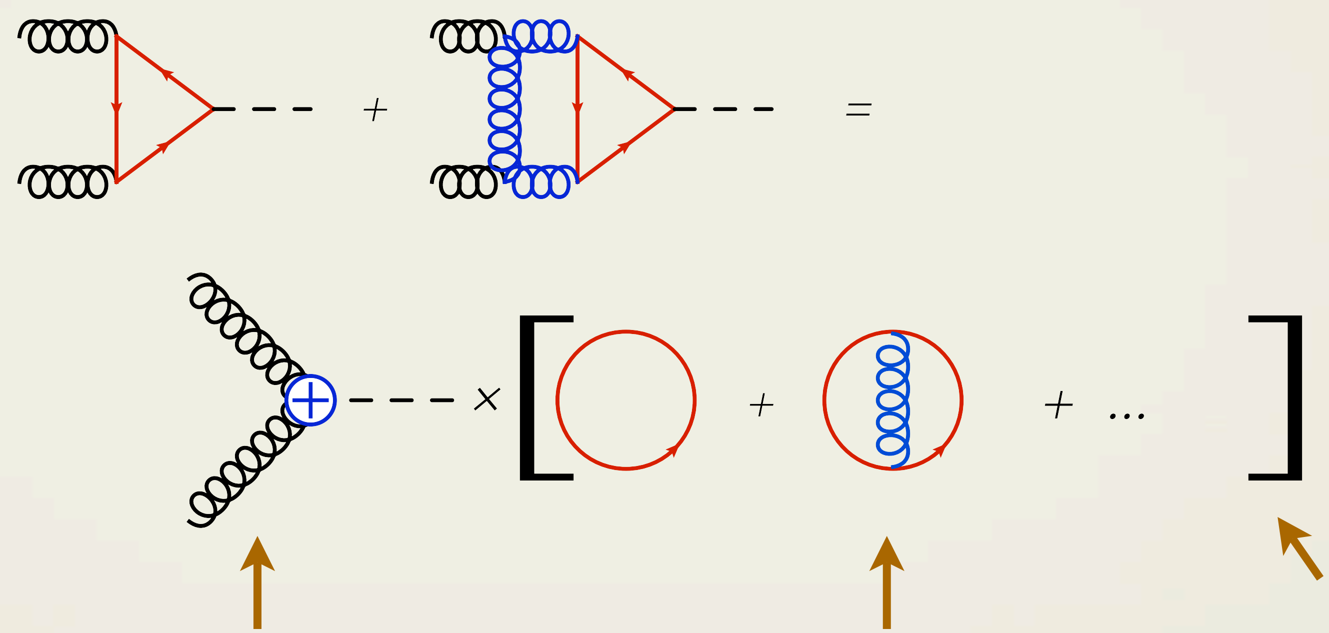
everything you ever wanted to know about Higgs physics: Djouadi, hep-ph/0503172, 173

Effective interactions

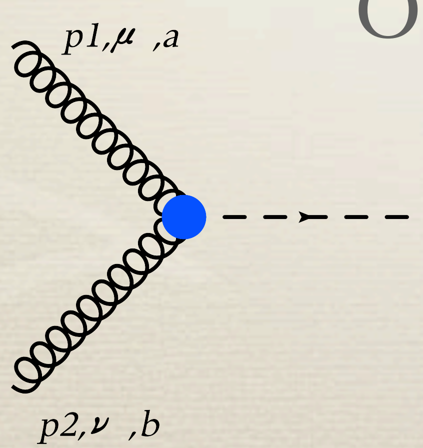
- Can get exact 2-loop NLO corrections without effective interaction (Djouadi, Graudenz, Spira, Zerwas 1995), but next term too tough

Effective field theory: exploit heavy mass of virtual particles

Two scales: M_{Higgs}, m_{top}



Only M_{Higgs} | Only m_{top} | $O(M^2_{Higgs}/4m^2_{top})$



$$= -i\frac{\alpha_s}{3\pi v}\left\{1+\frac{11}{4}\frac{\alpha_s}{\pi}\right\}\delta^{ab}\left[p_1\cdot p_2 g^{\mu\nu} - p_1^\nu p_2^\mu\right]$$

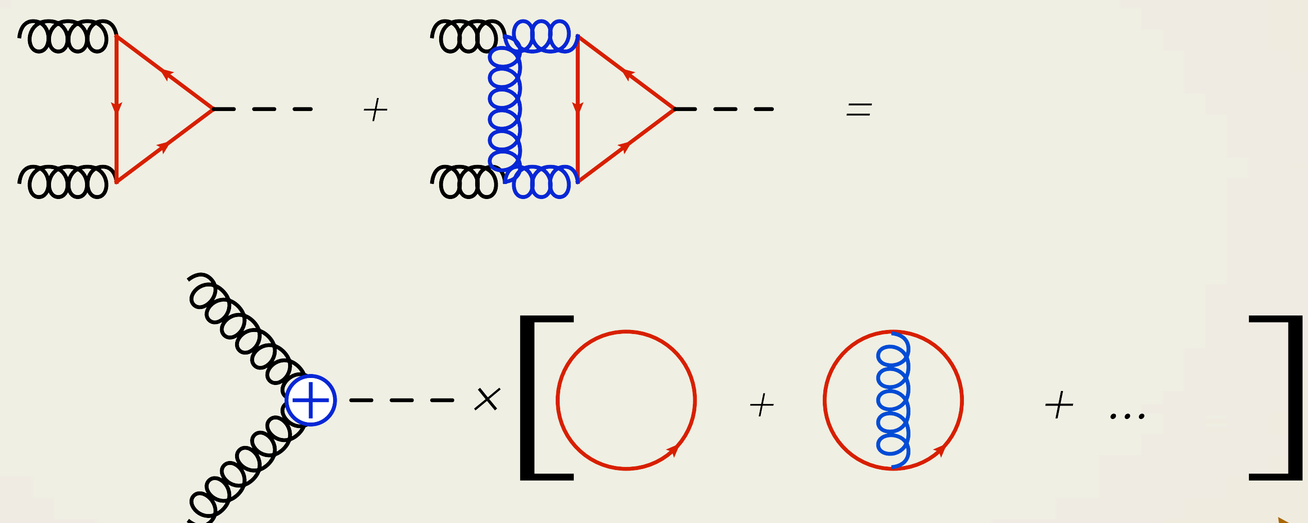
(also cubic, quartic gluon vertices)

Effective interactions

- Can get exact 2-loop NLO corrections without effective interaction (Djouadi, Graudenz, Spira, Zerwas 1995), but next term too tough

Effective field theory: exploit heavy mass of virtual particles

Two scales:
M_{Higgs}, m_{top}



Only M_{Higgs}

Only m_{top}

$O(M^2_{Higgs}/4m^2_{top})$

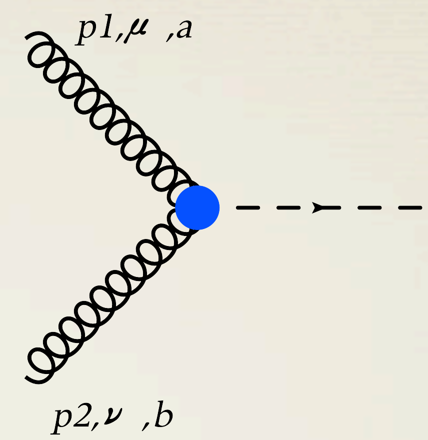
$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi}\left\{1 + \frac{11}{4}\frac{\alpha_s}{\pi}\right\}\frac{h}{v}F^a_{\mu\nu}F_a^{\mu\nu}$$

derived in backup slides

Tree-level

$(z = m_h^2/x_1 x_2 s)$

$$\sigma_{h_1 h_2 \to h} = \int dx_1\, dx_2 f_g(x_1) f_g(x_2)\, \hat{\sigma}(z) + \text{smaller partonic channels}$$



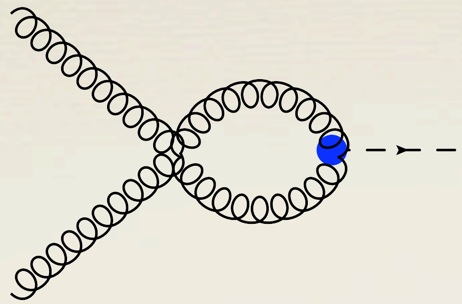
$$|\bar{\mathcal{M}}|^2 = \frac{1}{256(1-\epsilon)^2} \times |\mathcal{M}|^2 = \frac{\hat{s}^2}{576 v^2 (1-\epsilon)} \left(\frac{\alpha_s}{\pi}\right)^2$$

$$\frac{PS}{2\hat{s}} = \frac{\pi}{\hat{s}^2} \delta(1-z) \quad (\text{with } \hat{s} = x_1 x_2 s)$$

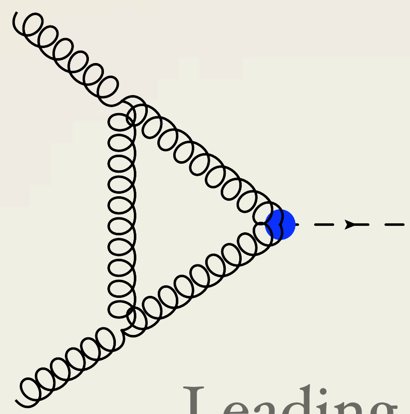
$$\hat{\sigma}_0(z) = \sigma_0\, \delta(1-z) = \frac{\pi}{576 v^2} \left(\frac{\alpha_s}{\pi}\right)^2 \delta(1-z)$$

Gluon-fusion: virtual

- <u>Virtual</u>:



$$= \sigma_0 \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{\hat{s}}{4\pi\mu^2}\right)^{-\epsilon}\left\{-\frac{13}{4\epsilon}-\frac{83}{12}\right\}\delta(1-z)$$



$$= \sigma_0 \frac{\alpha_s}{\pi}\Gamma(1+\epsilon)\left(\frac{\hat{s}}{4\pi\mu^2}\right)^{-\epsilon}\left\{-\frac{3}{\epsilon^2}+\frac{1}{4\epsilon}+\frac{47}{12}+2\pi^2\right\}\delta(1-z)$$

Leading soft+collinear singularity; emitting gluons from gluons gives color factor C_A=3

- <u>UV renormalization</u>: counterterm for α_s at leading order

Full d-dimensional LO

First term in beta-function

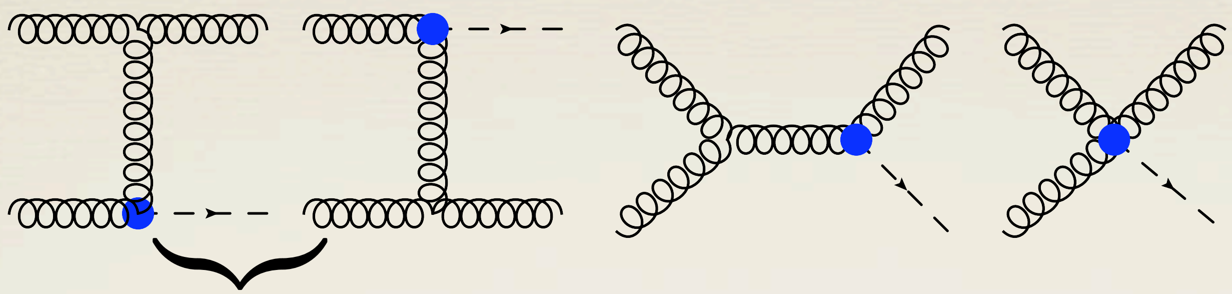
$$= \quad \hat{\sigma}_0^{(d)}(z)\frac{\alpha_s}{\pi}\frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}}\frac{1}{\epsilon}\left[-2\,b_0\right]$$

$$= \quad \sigma_0 \frac{\alpha_s}{\pi}\frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}}\left\{-\frac{11}{2}+\frac{N_F}{3}\right\}\left[\frac{1}{\epsilon}+1\right]\delta(1-z)$$

Number of light fermions

Gluon-fusion: real radiation

- Real:



singular

Phase space :

$$\frac{1}{2\hat{s}}\int \frac{d^d p_g}{(2\pi)^d}\frac{d^d p_H}{(2\pi)^d}\,(2\pi)\delta(p_g^2)\,(2\pi)\delta(p_H^2 - M_H^2)\,(2\pi)^d\delta^{(d)}(p_1 + p_1 - p_g - p_H)$$

$$= \frac{1}{16\pi\hat{s}}\frac{s^{-\epsilon}}{(4\pi)^{-\epsilon}\Gamma(1-e)}(1-z)^{1-2\epsilon}\int_0^1 d\lambda\,\lambda^{-\epsilon}(1-\lambda)^{-\epsilon}$$

$$\Rightarrow\quad \hat{t} = (p_1 - p_g)^2 = -\hat{s}(1-z)\lambda,\quad \hat{u} = (p_2 - p_g)^2 = -\hat{s}(1-z)(1-\lambda)$$

$$|\bar{\mathcal{M}}|^2 = 24\,\alpha_s\sigma_0\left\{\frac{(1-2\epsilon)}{(1-\epsilon)}\frac{M_H^8 + \hat{s}^4 + \hat{t}^4 + \hat{u}^4}{\hat{s}\hat{t}\hat{u}} + \frac{\epsilon}{2(1-\epsilon)^2}\frac{(M_H^4 + \hat{s}^2 + \hat{t}^2 + \hat{u}^2)^2}{\hat{s}\hat{t}\hat{u}}\right\}$$

$$\Downarrow$$

$$\Rightarrow (1-z)^{-1-2\epsilon}\lambda^{-1-\epsilon}(1-\lambda)^{-1-\epsilon}$$

singular

regulator

λ→0,1: collinear

z→1: soft

Real radiation and plus dists.

- Extract singularities using plus distribution expansion

$$\lambda^{-1-\epsilon} = -\frac{1}{\epsilon}\delta(\lambda) + \frac{1}{[\lambda]_+} - \epsilon\left[\frac{\ln\lambda}{\lambda}\right]_+ + \mathcal{O}(\epsilon^2), \quad \text{etc.}$$

$$\int_0^1 dx\, f(x)[g(x)]_+ = \int_0^1 dx\,[f(x)-f(0)]\,g(x)$$

$$= \sigma_0\frac{\alpha_s}{\pi}\left(\frac{\hat{s}}{4\pi\mu^2}\right)^{-\epsilon}\Bigg\{\overbrace{\left[\frac{3}{\epsilon^2}+\frac{3}{\epsilon}\right]\delta(1-z)}^{\text{cancels virtual poles}} - \frac{6}{\epsilon}\frac{1}{[1-z]_+} + \frac{6z(z^2-z+2)}{\epsilon}$$

1/Γ(1-ε)

$$+ \quad (3-\pi^2)\,\delta(1-z) - \frac{6}{[1-z]_+} + 12\left[\frac{\ln(1-z)}{1-z}\right]_+$$

$$- \quad 12z(z^2-z+2)\ln(1-z) - \frac{11}{2} + \frac{57z}{2} - \frac{45z^2}{2} + \frac{23z^3}{2}\Bigg\}$$

Remaining terms

- PDF renormalization: counterterm for initial-state collinear sings.

one for each PDF

$$= 2\,\hat{\sigma}_0^{(d)}(z)\frac{\alpha_s}{2\pi}\frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}}\frac{1}{\epsilon}P_{gg}(z)$$

subtracting P_{gg} from PDF same as adding to partonic cross section

$$= \sigma_0\frac{\alpha_s}{\pi}\frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}}\left\{\underbrace{\left(\frac{11}{2}-\frac{N_F}{3}\right)\delta(1-z)}_{\text{cancels UV counterterm}}+\underbrace{\frac{6}{[1-z]_+}-6z(z^2-z+2)}_{\text{cancels real radiation}}\right\}\left[\frac{1}{\epsilon}+1\right]$$

- Effective Lagrangian correction: $= \sigma_0\dfrac{\alpha_s}{\pi}\dfrac{11}{2}\delta(1-z)$

Gluon fusion: final result

- Arrive at the final NLO correction

$$\Delta\sigma = \sigma_0 \frac{\alpha_s}{\pi}\left\{\left(\frac{11}{2}+\pi^2\right)\delta(1-z) + 12\left[\frac{\ln(1-z)}{1-z}\right]_+ - 12z(-z+z^2+2)\ln(1-z) - \frac{11}{2}(1-z)^3 + 6\ln\frac{\hat{s}}{\mu^2}\left[\frac{1}{[1-z]_+} - z(z^2-z+2)\right]\right\}$$

($M^2/s \leq z \leq 1$) (integration over PDFs⇒integration over z)

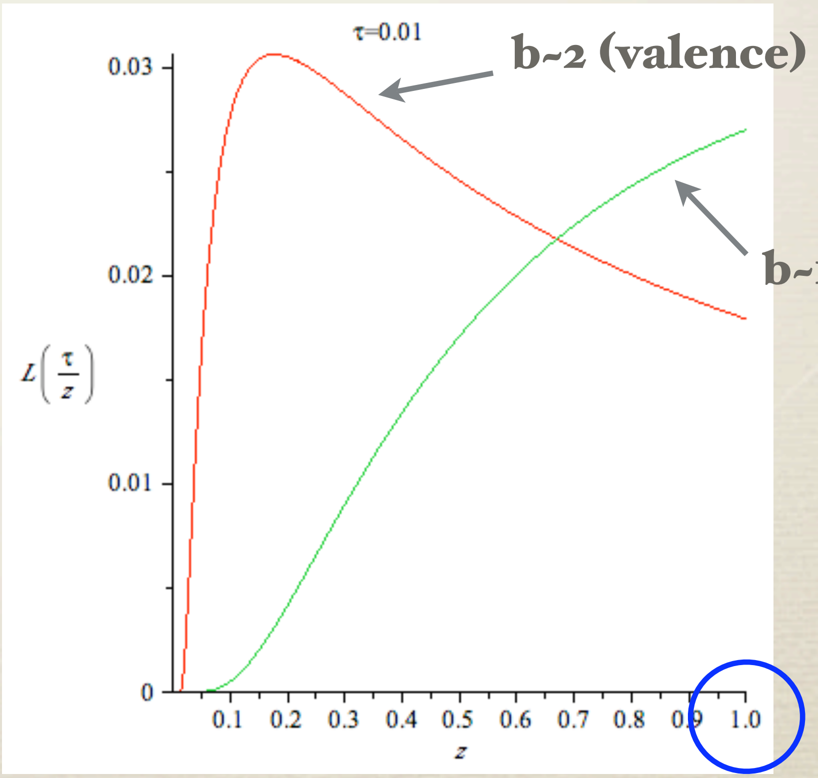
  - First source of large correction: $11/2+\pi^2 \Rightarrow$ 50% increase
  - Second source: shape of PDFs enhances *threshold* logarithm

$$\sigma_{had} = \tau\int_\tau^1 dz\,\frac{\sigma(z)}{z}\,\mathcal{L}\left(\frac{\tau}{z}\right)$$

$$\mathcal{L}(y) = \int_y^1 dx\,\frac{y}{x}\,f_1(x)f_2(y/x) \quad \text{(partonic luminosity)}$$

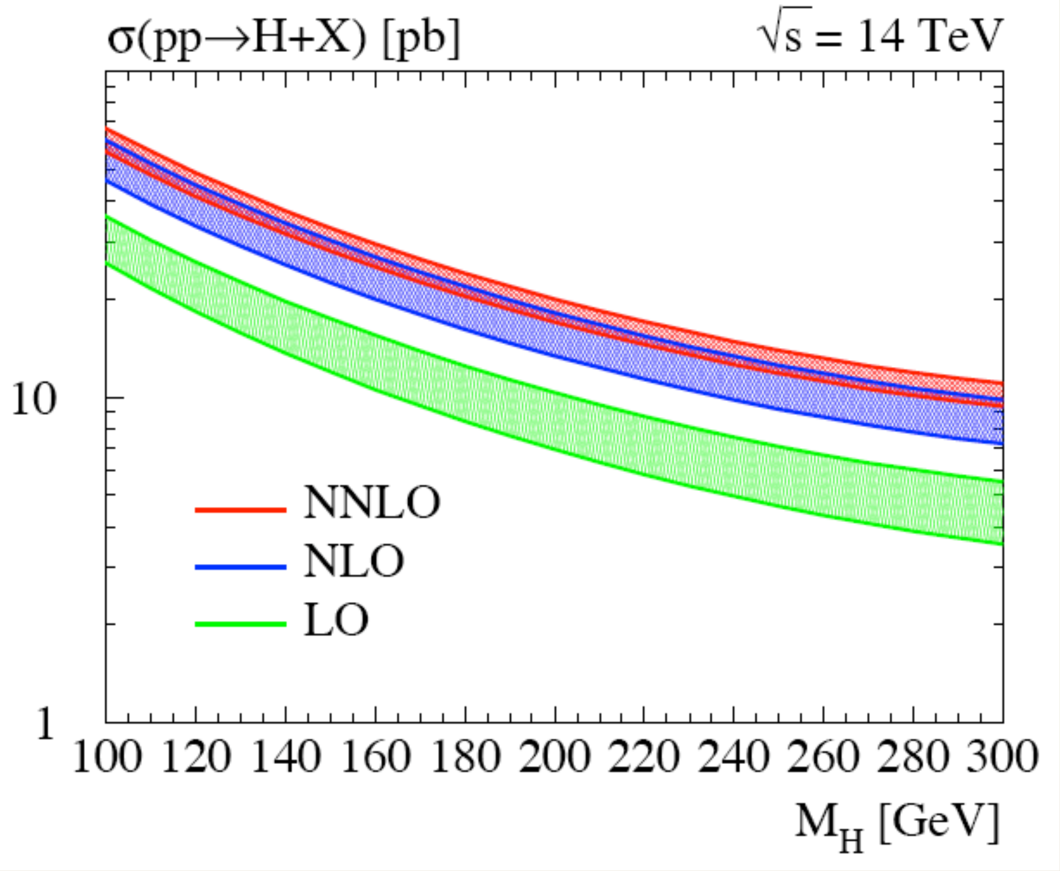
Assume $f_i \sim (1-x)^b$; plot L for various b
Look for peak near $z \approx 1$

⇒Sharp fall-off of gluon PDF enhances correction



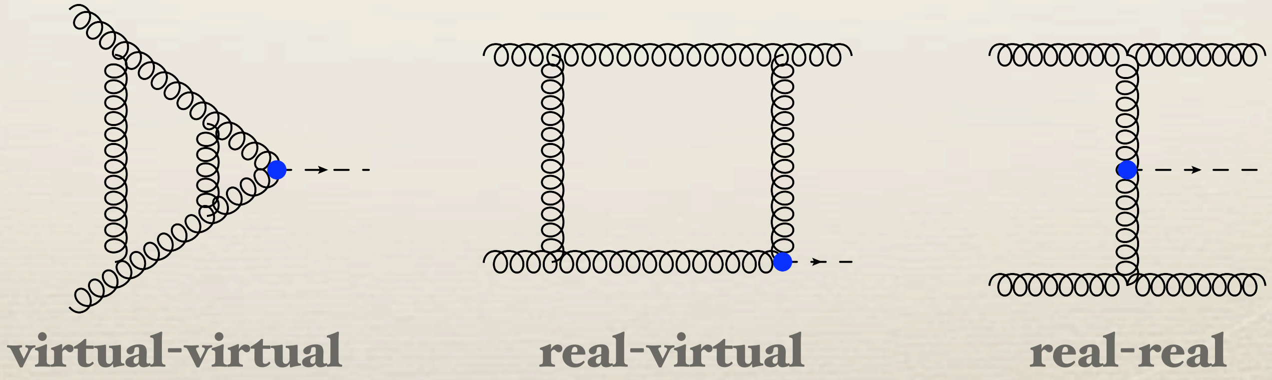
Inclusive Higgs at NNLO: scale variation

- Full calculation at NNLO in the EFT



- Scale variation, especially at LO, can badly underestimate error!

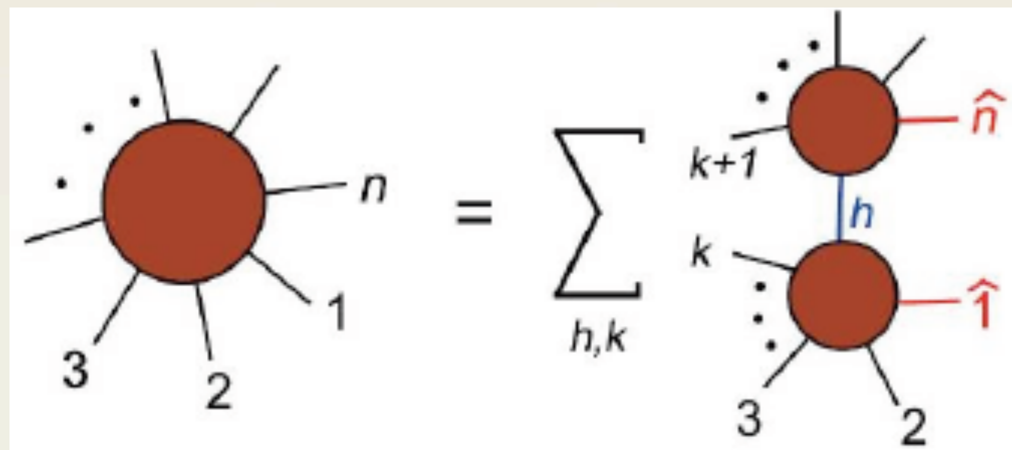
Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith van Neerven '03



Survey of Current Topics

Recursion relations

- Can go to high multiplicity at LO using *recursion relations* rather than diagrams (Berends-Giele, Cachazo-Svrcek-Witten, Britto-Cachazo-Feng)



$pp \to n$ jets gluons only	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
MC cross section [pb]	$8.915 \cdot 10^7$	$5.454 \cdot 10^6$	$1.150 \cdot 10^6$	$2.757 \cdot 10^5$	$7.95 \cdot 10^4$
stat. error	0.1%	0.1%	0.2%	0.5%	1%
	integration time for given stat. error [s]				
CSW (HAAG)	4	165	1681	12800	$2 \cdot 10^6$
CSW (CSI)	-	480	6500	11900	197000
AMEGIC (HAAG)	6	492	41400	-	-
COMIX (RPG)	159	5050	33000	38000	74000
COMIX (CSI)	-	780	6930	6800	12400

Feynman diagrams → AMEGIC (HAAG)

Berends-Giele → COMIX (RPG)

Tab. 4 Cross section and evaluation times for different matrix element (phase space) generation methods for multi-gluon scattering at the LHC, given in pb. Numbers were generated on a 2.53 GHz Intel® Core™2 Duo T9400 CPU. For cuts and parameter settings, cf. Tab. 3.

Gleisberg et al., 0808.3672

Merging LO with PS

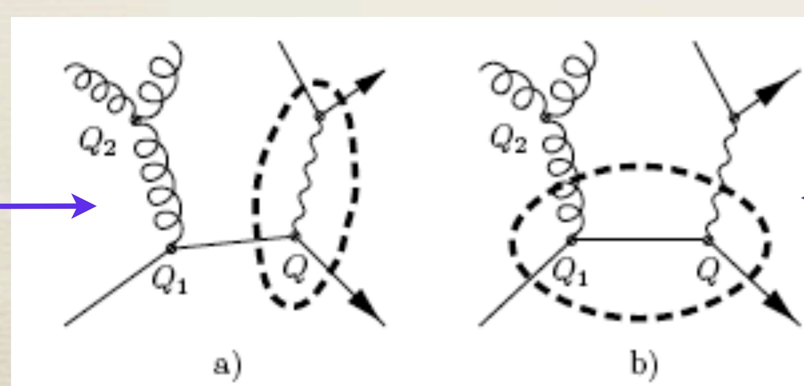
- ☑ Want to attach parton shower: describes soft/collinear jets, very high multiplicity allows connections to hadronization
- ☑ Don't want to double count emissions from diagrams and PS!

CKKW matching (for W+jets):

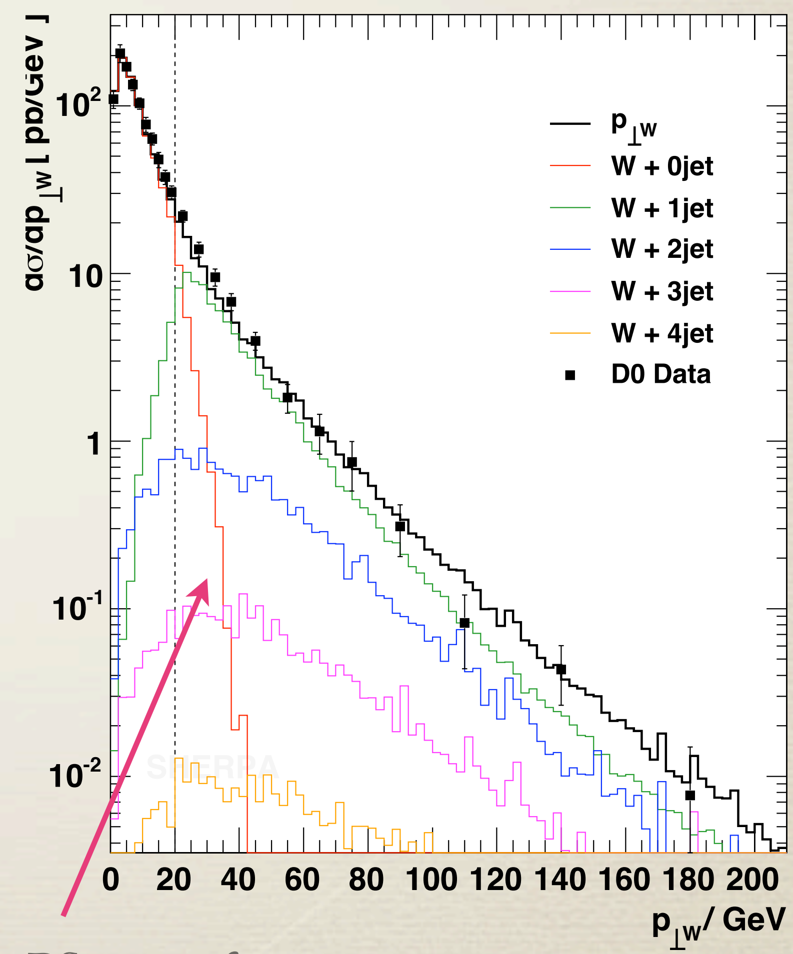
(Catani, Krauss, Kuhn, Webber hep-ph/0109231)); also MLM matching (Mangano)

- Define jet resolution parameter Q_{cut}
- Select W+n jet process according to $P_n = \dfrac{\sigma_n}{\sum_i \sigma_i}$
- Generate shower starting from this configuration
- Reweight internal lines with Sudakov factor
- Veto emissions above Q_{cut}

Shower from W+0 jets



Shower from W+1 jet



Pure PS too soft

Krauss et al., hep-ph/0409106

Available NLO results

- Corrections can be surprisingly large (time-like π^2, phase-space edges) ⇒ should have NLO for all processes, what is known?
- Roughly: 2→2 known (review of techniques: Denner 0709.1075), 2→3 challenging (spurious singularities, algebraic complexity) but doable, very few 2→4 results known

Partial listing at http://www.cedar.ac.uk/hepcode/

Some examples:

- MCFM (Campbell, Ellis): V+≤2 jets, VH, H+≤1 jet, QQ
- NLOJET++ (Nagy): ≤3 jets
- DIPHOX (Aurenche et al.): γγ, γ+jet
- VBFNLO (Arnold et al.): many vector-boson fusion signals, backgrounds
- ...

process	$\sigma_{NLO,NNLO}$ (by)
$gg \to H$ HIGLU MCFM MC@NLO,POWHEG	S.Dawson, NPB 359 (1991), A.Djouadi, M.Spira, P.Zerwas, PLB 264 (1991) C.J.Glosser *et al.*, JHEP (2002); V.Ravindran *et al.*, NPB 634 (2002) D. de Florian *et al.*, PRL 82 (1999) R.Harlander, W.Kilgore, PRL 88 (2002) (NNLO) C.Anastasiou, K.Melnikov, NPB 646 (2002) (NNLO) V.Ravindran *et al.*, NPB 665 (2003) (NNLO) S.Catani *et al.* JHEP 0307 (2003) (NNLL) G.Bozzi *et al.*, PLB 564 (2003), NPB 737 (2006) (NNLL) C.Anastasiou, R.Boughezal, F.Petriello, JHEP (2008) (QCD+EW)
$q\bar{q} \to (W,Z)H$	T.Han, S.Willenbrock, PLB 273 (1991) O.Brien, A.Djouadi, R.Harlander, PLB 579 (2004) (NNLO)
$q\bar{q} \to q\bar{q}H$	T.Han, G.Valencia, S.Willenbrock, PRL 69 (1992) T.Figy, C.Oleari, D.Zeppenfeld, PRD 68 (2003)
$q\bar{q}, gg \to t\bar{t}H$	W.Beenakker *et al.*, PRL 87 (2001), NPB 653 (2003) S.Dawson *et al.*, PRL 87 (2001), PRD 65 (2002), PRD 67,68 (2003)
$q\bar{q}, gg \to b\bar{b}H$	S.Dittmaier, M.Krämer, M.Spira, PRD 70 (2004) S.Dawson *et al.*, PRD 69 (2004), PRL 94 (2005)
$gb(\bar{b}) \to b(\bar{b})H$ MCFM	J.Cambell *et al.*, PRD 67 (2003)
$b\bar{b} \to (b\bar{b})H$ MCFM	D.A.Dicus *et al.* PRD 59 (1999); C.Balasz *et al.*, PRD 60 (1999). R.Harlander, W.Kilgore, PRD 68 (2003) (NNLO)

process	$\sigma_{NLO,NNLO}$ (by)
$W,Z(\to l\nu, ll)$ MCFM MC@NLO,POWHEG ResBos	W.L.van Neerven *et al*, NBP 382 (1992) R.Hamberg, W.L.van Neerven and T.Matsuura, NPB 359 (1991) (NNLO) C.Anastasiou, L.Dixon, K.Melnikov, F.Petriello (NNLO, distrib.) C.Balazs, C.-P. Yuan, PRD 56 (1997) (resummed NLO)
WW, ZZ, WZ AYLEN/EMILIA MCFM MC@NLO,POWHEG	J.Ohnemus *et al.*, PRD 44 (1991); PRD 43 (1991); PRD 50 (1994) B.Mele *et al.*, NPB 357 (1991) S.Frixione *et al.*, NPB 410 (1993); NPB 383 (1992) L.Dixon *et al.*, NPB 531 (1998); PRD 60 (1999) J.Campbell, R.K.Ellis, F.Tramontano, PRD 60 (1999)
VVV VBFNLO	V.Hankele, D.Zeppenfeld, PLB (2007); F.Campanario *et al.* PRD (2008) A.Lazopoulos, K.Melnikov, F.Petriello, PRD 76 (2007) T.Binoth *et al.* JHEP 0806.082 (2008)
$W,Z+\leq 2j$ MCFM	W.Giele, N.Glover, D.Kosower, NPB 403 (1993) J.Campbell *et al*, PRD 65 (2002); PRD 68 (2003)
$W,Z+3j$	C.Berger *et al.* (Blackhat collaboration), arXiv:00902.2760 R.K.Ellis *et al.* JHEP 0901:012, 2009.
$WW+j$	J.Campbell, R.K.Ellis, G.Zanderighi, JHEP 0712:056 (2007) S.Dittamier, S.Kallweit, P.Uwer, PRL 100 (2008)
$W,Z+Q$ MCFM	W.Giele *et al.*, PLB 372 (1996); E.Berger *et al.*, PRD 54 (1996); M.Aivazia *et al*, PRD 50 (1994); J.Collins, PRD 58 (1998); T.Stelzer *et al.*, PRD 56 (1997); J.Campbell, *et al.*, PRD 69 (2004)
$W,Z+Q\bar{Q}$ MCFM	J.Campbell, R.K.Ellis, PRD 62 (2000) ($m_Q \to 0$) F.Maltoni *et al.*, hep-ph/0505014 ($m_Q \to 0$) Febres Cordero *et al.*, PRD 74 (2006), PRD 78 (2008), arXiv:0906.1923.

process	$\sigma_{NLO,NNLO}$ (by)
$Q\bar{Q}$ MCFM MC@NLO,POWHEG	P.Nason, S.Dawson, R.K.Ellis, NPB 303 (1988); NPB 327 (1989) W.Beenakker *et al.*, PRD 40 (1989); NPB 351 (1991) M.Mangano. P.Nason, G.Ridolfi, NPB 373 (1992) R.Bonciani, S.Catani, M.L.Mangano, P.Nason, NPB 529 (1998) (NNL) N.Kidonakis, R.Vogt, Eur. Phys. J. C 33 (2004), C 36 (2004) (≃NNLO) N. Kidonakis, Mod. Phys. Lett. A 19 (2004) (NNNLL+NNLO) A.Banfi, E.Laenen, PRD 71 (2005) and refs. therein (NLL+NLO) W.Bernreuther *et al.*, NPB 690 (2004) (spin correlations) M.Czakon, A.Mitov, S.Moch, PLB 651 (2007), NPB 798 (2008), arXiv:0811.4119 (2-loop NNLO)
$Q\bar{Q}$+j	S.Dittmaier, P.Uwer, S. Weinzierl, PRL 98:262002 (2008)
$t\bar{t}+b\bar{b}$	A.Bredenstein, A.Denner, S.Dittmaier, S.Pozzorini, arXiv:0905.0110
single top MCFM MC@NLO	M.Smith, S.Willenbrock, PRD 54 (1996) G.Bordes, B.van Eijk, NPB 435 (1995) T.Stelzer *et al.*, PRD 56 (1997) B.W.Harris *et al.*, PRD 66 (2002) Z.Sullivan, PRD 70 (2004) J.Campbell, R.K.Ellis, PRD 70 (2004) Q.-H. Cao *et al*, PRD 71 (2005); hep-ph/0504230
$pp(\bar{p}p) \to \leq 3j$ NLOJET++ JETRAD	W.Giele, N.Glover, D.Kosower, NPB 403 (1993) Z.Kunszt and D.Soper, PRD 46 (1992) W.Kilgore and W.Giele, PRD 55 (1997) Z.Nagy, PRL88 (2002), PRD 68 (2003) (3j)

from L. Reina

NLO difficulties

- Techniques known to handle real radiation contributions

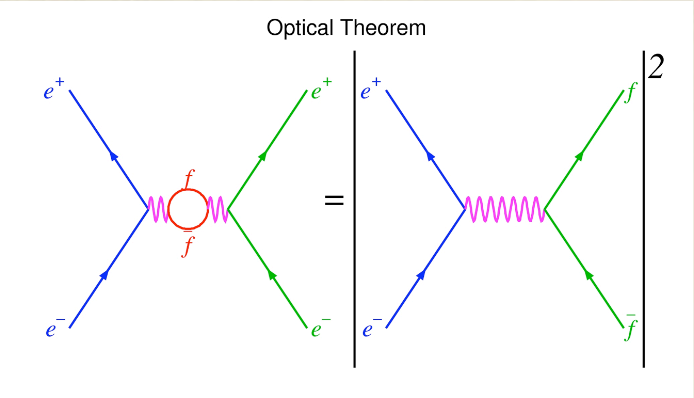
  already discussed *phase-space slicing*

  Dipole subtraction: construct approximations that reproduce full QCD in singular limits, are analytically integrable (dipoles); cancel poles, numerically integrate full QCD - dipoles (Catani, Seymour hep-ph/9605323)

- Hard part are (were?) the loops for 2→3 and beyond...

Factorial growth of diagrams and enormous algebraic expressions, final results often simpler then intermediate steps ⇒ better organizing principle?

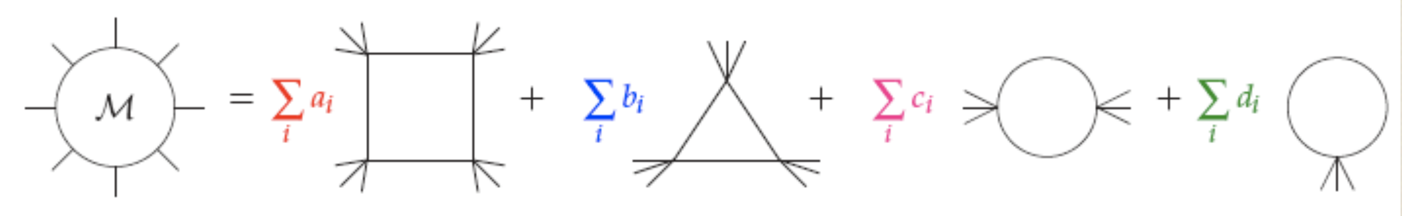
Unitarity and NLO amplitudes



Put loop propagators on-shell (“cut” them) to get imaginary parts from trees

Some success using this+singular limits to construct loops from trees for multi-leg processes Bern, Dixon, Dunbar, Kosower, 1990s

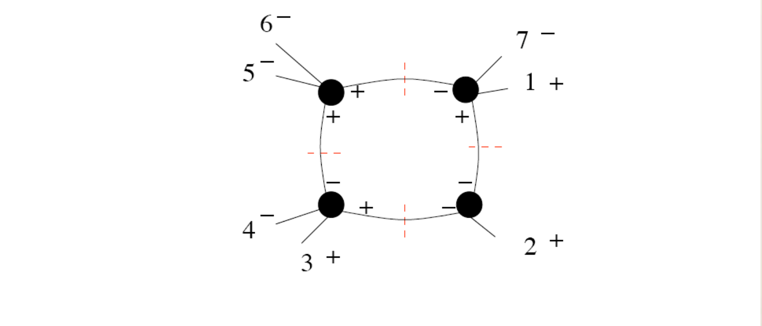
Can decompose 1-loop amplitudes into basis of scalar integrals:



$$\mathcal{M} = \sum_i a_i \,\square + \sum_i b_i \,\triangle + \sum_i c_i \,\bigcirc + \sum_i d_i \,\bigcirc$$

Try to isolate box coefficients a_i by cutting 4 propagators

Only find a solution for *complex* momenta Britto, Cachazo, Feng 2004



$$\text{coeff} = \frac{1}{2}\frac{[\ell_1\,\ell_4]^3}{[\ell_1\,2][2\,\ell_4]}\frac{[4\,\ell_2]^3}{[\ell_2\,\ell_1][\ell_1\,3][3\,4]}\frac{[5\,6]^3}{[6\,\ell_3][\ell_3\,\ell_2][\ell_2\,5]}\frac{[\ell_3\,7]^3}{[7\,1][1\,\ell_4][\ell_4\,\ell_3]}$$

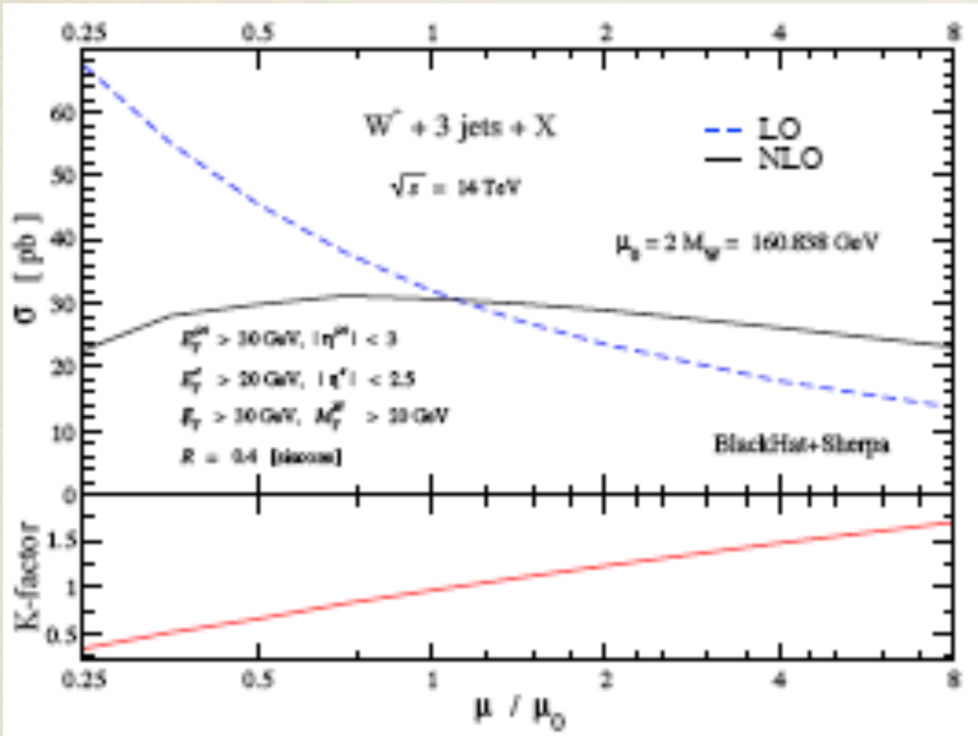
$$= -\frac{\langle 1\,2\rangle^3\langle 2\,3\rangle^3[5\,6]^3}{\langle 7\,1\rangle\langle 3\,4\rangle\langle 2|P_{3,4}|5]\langle 2|P_{7,1}|6]\langle 2|P_{3,4}P_{5,6}|7\rangle\langle 2|P_{7,1}P_{5,6}|4\rangle}$$

$$\frac{1}{2}\sum_{\mathcal{S}} A_1^{\text{tree}} A_2^{\text{tree}} A_3^{\text{tree}} A_4^{\text{tree}}$$

No 1-loop diagrams! Just compute tree graphs... and we know recursive techniques, can do numerically

2→4 at NLO

W+3 jets



Unitarity-based approach

Large N_C: Rocket

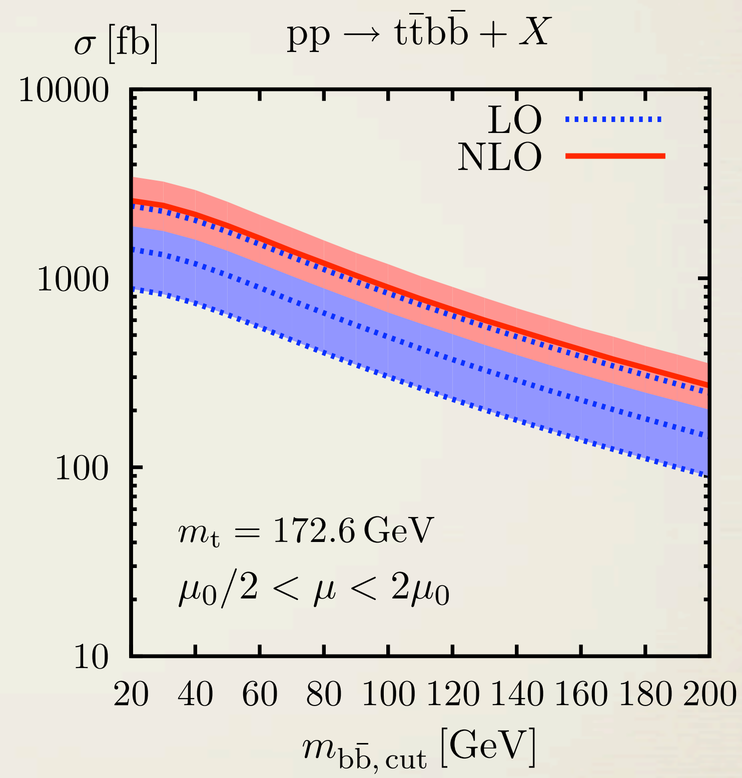
0906.1445, R. K. Ellis, Melnikov, Zanderighi

Full QCD: Blackhat

0907.1984, Berger, Bern, Dixon, Cordero, Forde, Gleisberg, Ita, Kosower, Maitre

(Even preliminary W+4 jets from Blackhat)

ttbb: background to ttH, important for bottom Yukawa measurement



Traditional Feynman diagrams

Bredenstein, Denner, Dittmaier, Pozzorini 0905.0110

Merging NLO with PS

- Want to combine NLO with parton shower ⇒ first hard emission described by NLO calculation, loops give right normalization
- Need to avoid double counting real-emission corrections
- Two working programs: MC@NLO (Frixione, Webber), POWHEG (Frixione, Nason, Oleari)

Correct at low p_T

Matches to NLO at high p_T

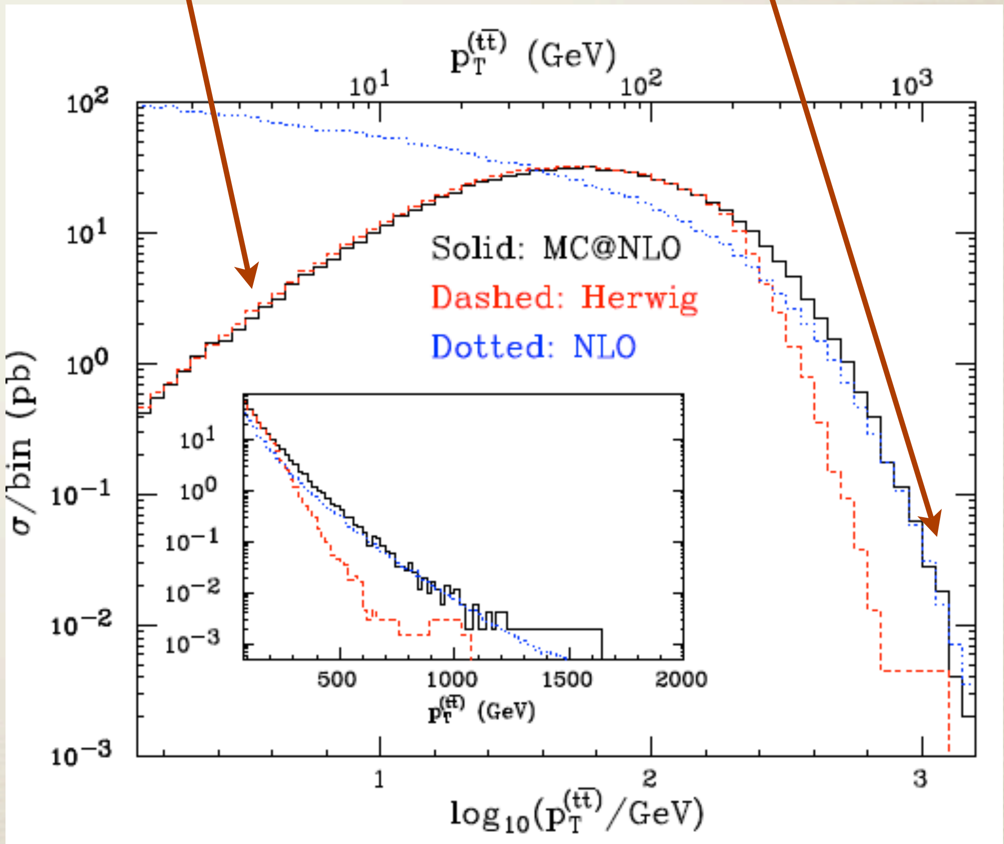
$$d\sigma_{\mathrm{POWHEG}} = \overline{B}(\Phi_n)\, d\Phi_n \left\{ \Delta(\Phi_n, p_T^{\min}) + \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \Delta(\Phi_n, p_T)\, d\Phi_r \right\}$$

$$\overline{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r \left[R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r) \right]$$

$$\Delta(\Phi_n, p_T) = \exp\left[-\int d\Phi'_r \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta\left(k_T(\Phi_n, \Phi'_r) - p_T\right) \right]$$

Virtual corrections included together with counterterms

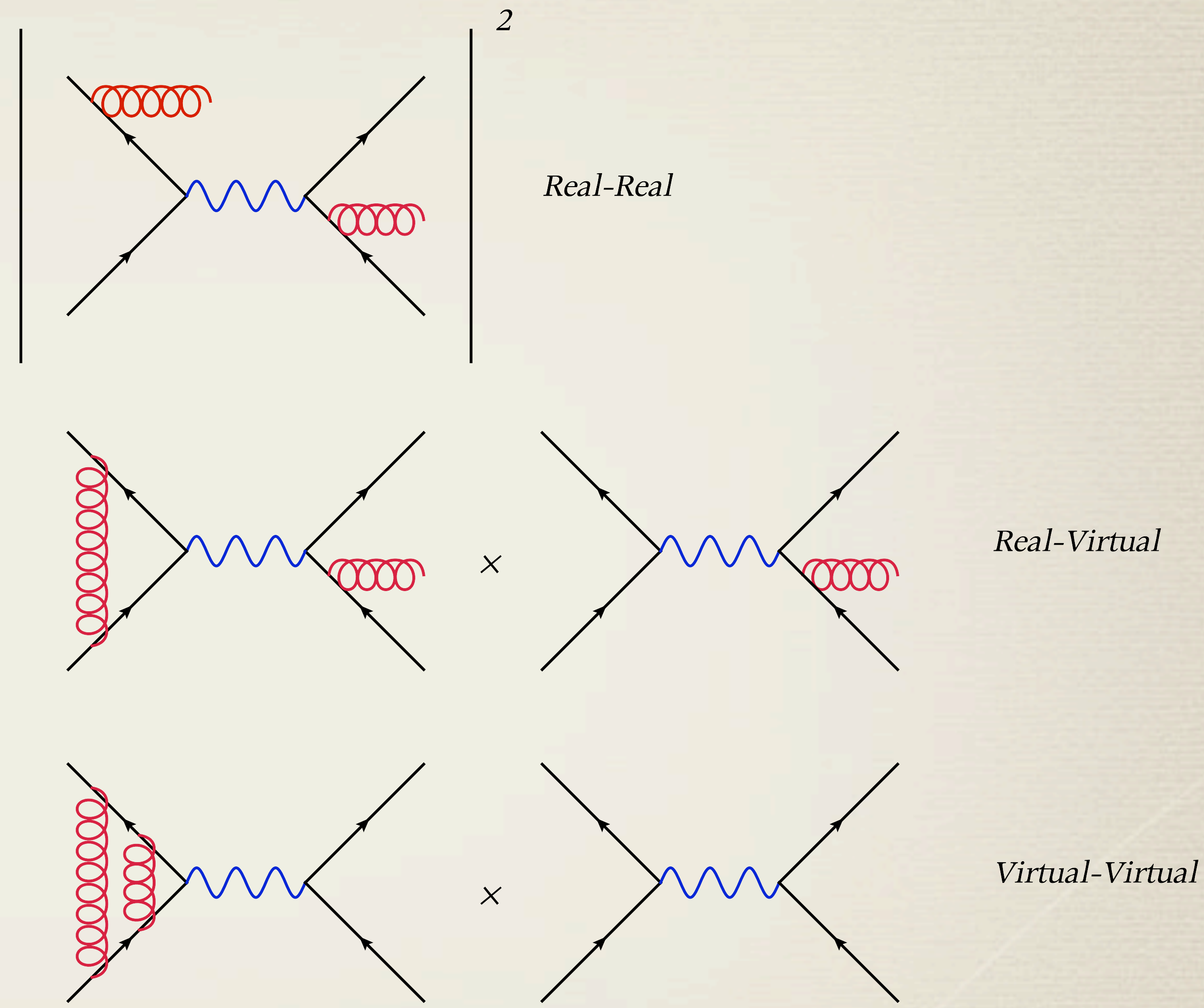
full real radiation in modifed Sudakov factor



Correct normalization to $O(\alpha_s)$, matches to NLO hard emission at high p_T, and shower at low p_T

Computing σ: NNLO

$$\sigma = \overbrace{\sigma_0}^{LO} + \overbrace{\frac{\alpha_s}{\pi}\sigma_1}^{NLO} + \overbrace{\left(\frac{\alpha_s}{\pi}\right)^2 \sigma_2}^{NNLO} + \ldots$$



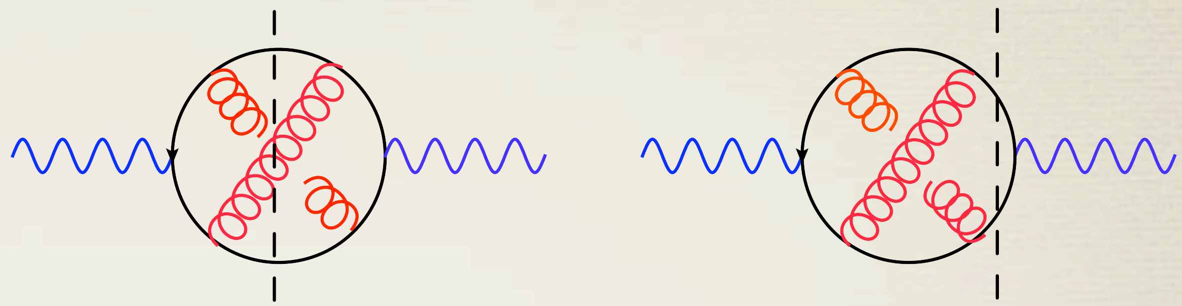
When is NNLO necessary?

- ☑ When NLO corrections are large, and NNLO is needed to check expansion (gg→H)
- ☑ For benchmark processes where high precision is needed (DIS, Drell-Yan for PDFs, $e^+e^- \to 3$ jets for α_s)

Integration-by-parts

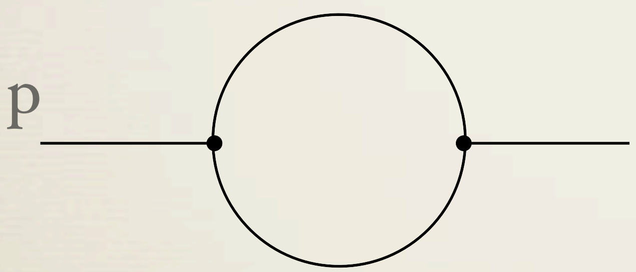
- Use optical theorem, map to the calculation of loop integrals

σ(γ*→hadrons)=Im(γ*→ γ*)/s



Real-Real cut Virtual-Virtual cut

- Integration-by-parts to reduce loops integrals to a few "master integrals" Chetyrkin, Tkachov 1981



$$\mathcal{I}(\nu_1,\nu_2) = \int d^dk\,\frac{1}{k^{2\nu_1}(k+p)^{2\nu_2}}$$

Set $\int d^dk\,\frac{\partial}{\partial k^\mu}\left[\frac{k^\mu}{k^{2\nu_1}(k+p)^{2\nu_2}}\right] = 0$

Derive $(d-2\nu_1-\nu_2)\mathcal{I}(\nu_1,\nu_2) - \nu_2\mathcal{I}(\nu_1-1,\nu_2+1) + \nu_2 p^2\mathcal{I}(\nu_1,\nu_2+1) = 0$

Apply to $\mathcal{I}(1,1) \Rightarrow \mathcal{I}(1,2) = -\frac{d-3}{p^2}\mathcal{I}(1,1)$

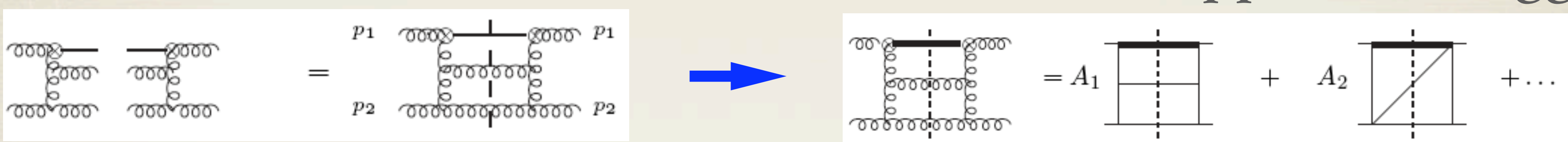
$\Rightarrow$ algebraically relate different integrals

$$\begin{aligned} R^{\overline{\mathrm{MS}}}(s) = 3\sum_f Q_f^2\Big(1 &+ \bar{\alpha}_s/\pi + (\bar{\alpha}_s/\pi)^2\{+\tfrac{365}{24} - 11\zeta(3) - N_f[\tfrac{11}{12} - \tfrac{2}{3}\zeta(3)]\} \\ &+ (\bar{\alpha}_s/\pi)^3\{+\tfrac{87029}{288} - \tfrac{1103}{4}\zeta(3) + \tfrac{275}{6}\zeta(5) + N_f[-\tfrac{7847}{216} + \tfrac{262}{9}\zeta(3) - \tfrac{25}{9}\zeta(5)] \\ &+ N_f^2[+\tfrac{151}{162} - \tfrac{19}{27}\zeta(3)] - \pi^2/48(11 - \tfrac{2}{3}N_f)^2\} + O(\alpha_s^4)\Big) \\ &+ \Big(\sum_f Q_f\Big)^2(\bar{\alpha}_s/\pi)^3[\tfrac{55}{72} - \tfrac{5}{3}\zeta(3)] + O(\alpha_s^4)\,. \end{aligned}$$

Gorishny, Kataev, Larin 1988; Surguladze, Samuel 1991

1,2-scale problems

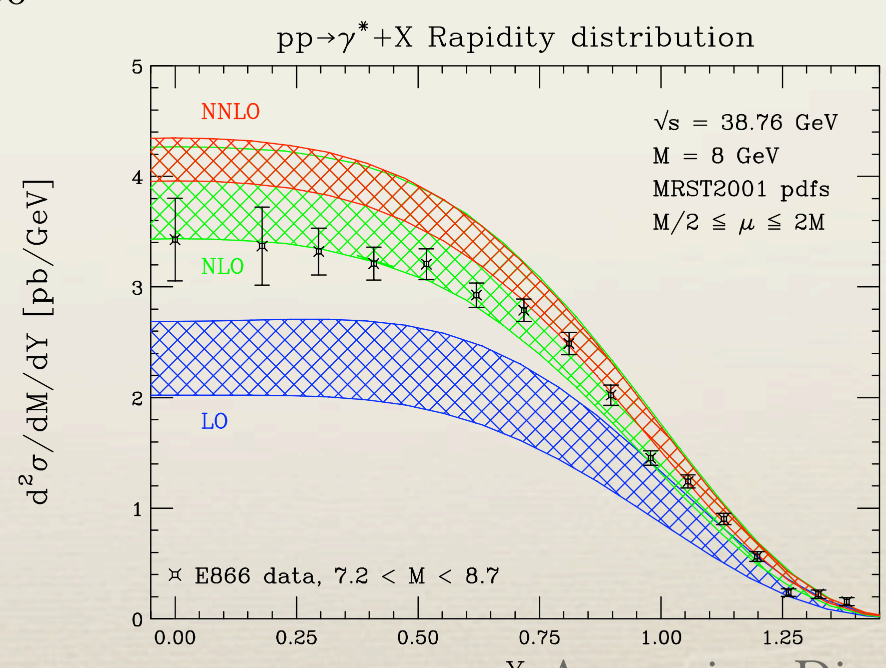
- Same IBP technology can be applied to hadron collider cross sections (Anastasiou, Melnikov hep-ph/0207004) ⇒ first applied to Higgs

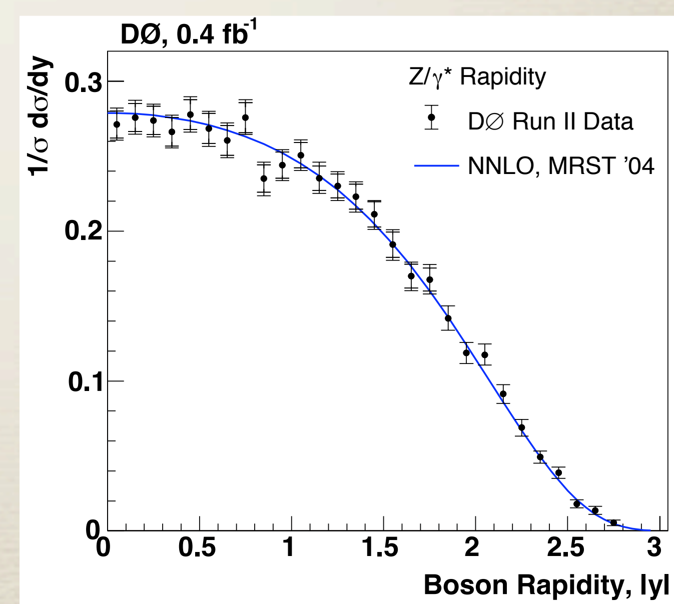


- W, Z rapidity distributions: depend on M²/s and Y ⇒ introduce a fictitious particle to allow use of IBP with rapidity constraint

$$\overbrace{\delta\left(\frac{p_V\cdot p_1}{p_V\cdot p_2}-u\right)}^{\text{phase-space constraint}} \rightarrow \overbrace{\frac{p_V\cdot p_2}{p_V\cdot(p_1-up_2)-i0}}^{\text{fictitious propagator}} - \text{c.c.} \qquad \left(u=\frac{x_1}{x_2}e^{-2Y}\right)$$

Important constraint on PDFs from fixed-target scattering (high-x quarks)





Anastasiou, Dixon, Melnikov, FP 2003

Fully differential NNLO

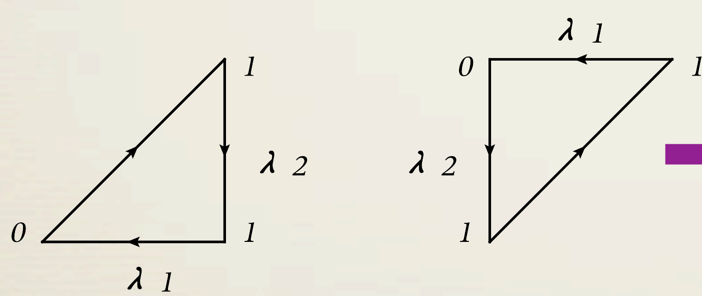
- Desirable to account fully for experimental constraints
- How to arrange singularity cancellation between real and virtual graphs for numerical integration?

Utilize regulators in explicit phase-space parametrizations

$$d\Pi_E = N\int_0^1 d\lambda_1 d\lambda_2 d\lambda_3 d\lambda_4 [\lambda_1(1-\lambda_1)]^{1-2\epsilon}[\lambda_2(1-\lambda_2)]^{-\epsilon}[\lambda_3(1-\lambda_3)]^{-\epsilon} \times [\lambda_4(1-\lambda_4)]^{-\epsilon-1/2} D^{2-d},$$

"Entangled" singularities: $\mathcal{I} = \int_0^1 dx\, dy\, \frac{\lambda_1^\epsilon \lambda_2^\epsilon}{(\lambda_1+\lambda_2)^2}$

Anastasiou, Melnikov, FP 2003-2004 for Higgs, W, Z



$$\mathcal{I} = \int_0^1 dx\, dy\, \frac{\lambda_1^{-1+2\epsilon}\lambda_2^\epsilon}{(1+\lambda_2)^2} + \int_0^1 dx\, dy\, \frac{\lambda_2^{-1+2\epsilon}\lambda_1^\epsilon}{(1+\lambda_1)^2}$$

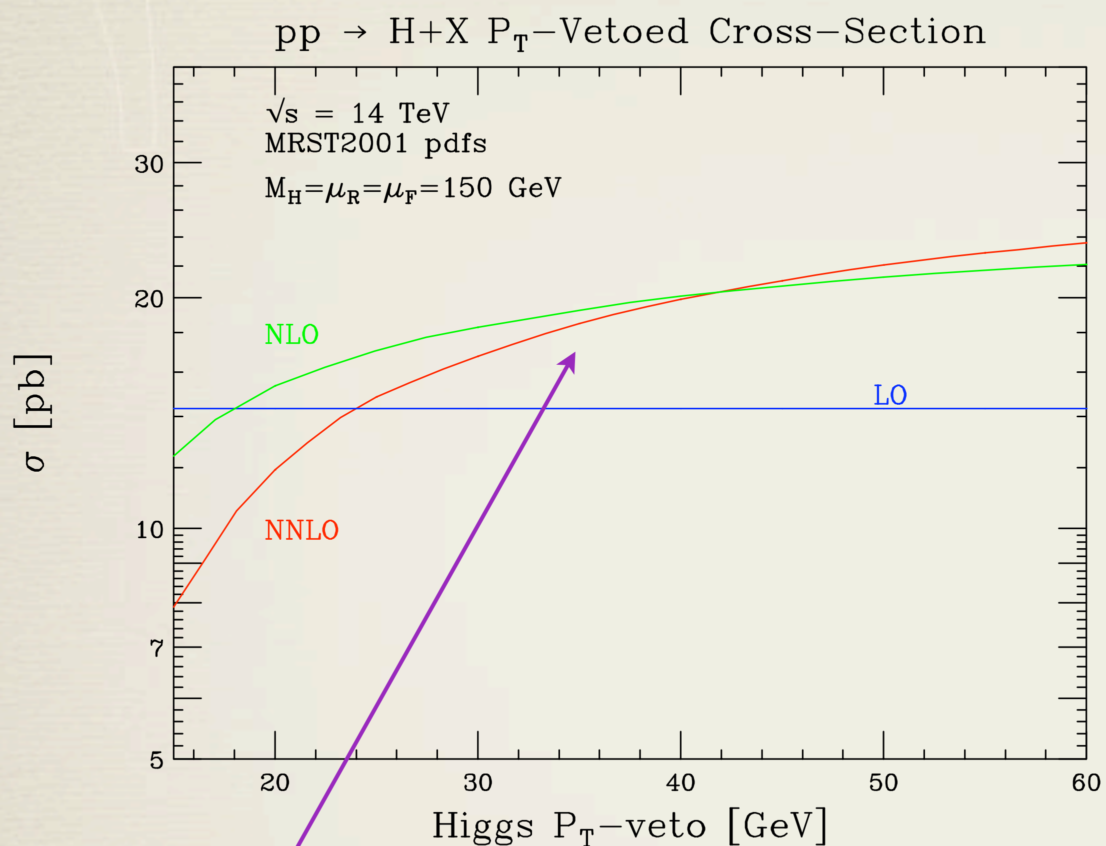
Use singular structure of QCD to build analytically-integrable subtraction terms

Gehrmann, Gehrmann de-Ridder, Glover 2004-2007 for e+e-→3 jets; Catani, Grazzini 2007 for Higgs; many others

$$\begin{aligned}
\mathrm{d}\sigma_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^R_{NNLO} - \mathrm{d}\sigma^S_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\
&+ \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^S_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^{V,2}_{NNLO}
\end{aligned}$$

Phenomenology at NNLO

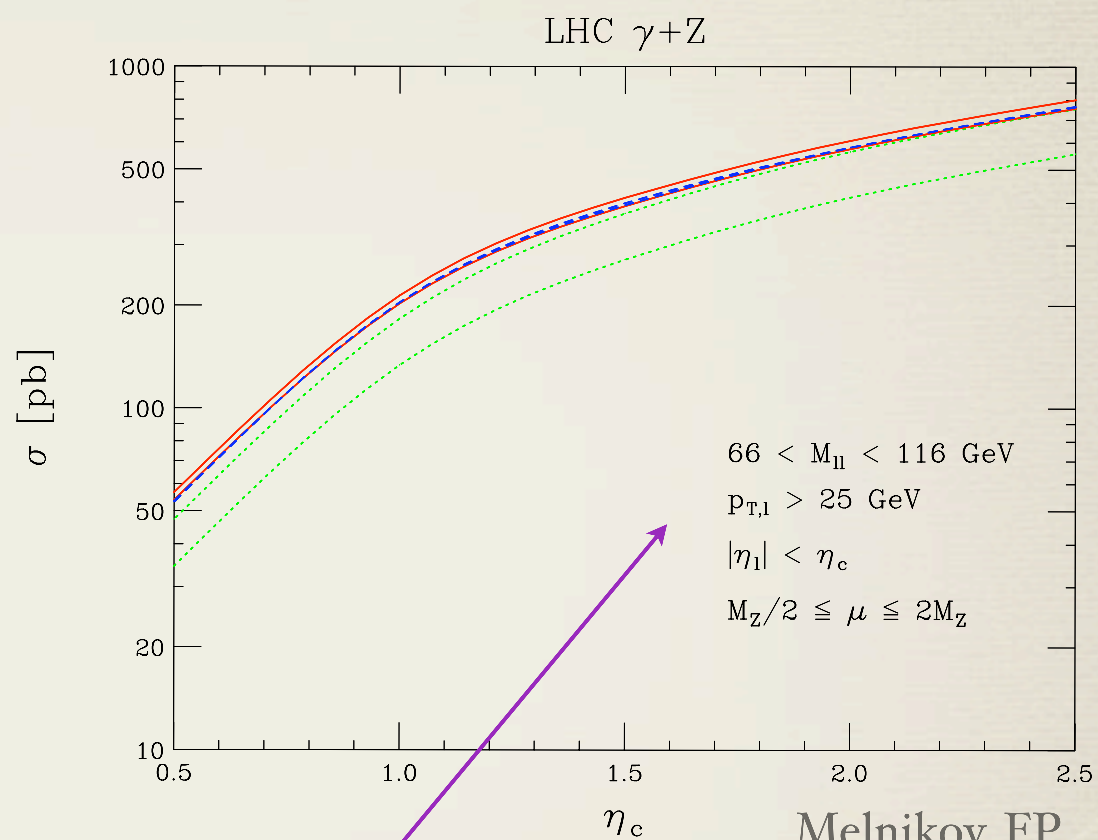
Higgs at LHC:



Anastasiou, Melnikov, FP hep-ph/0501130

NNLO corrections have kinematic dependence!

W,Z at LHC:



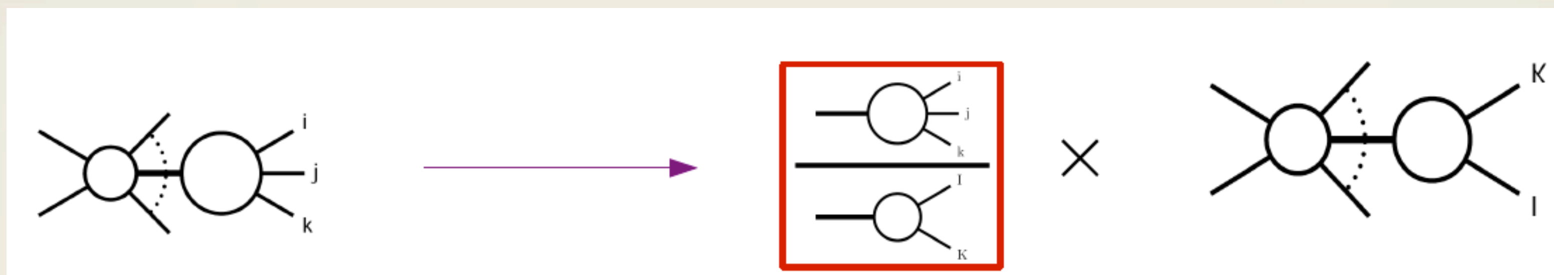
Melnikov, FP hep-ph/0609070

Include acceptance cuts, spin correlations for percent-level "partonic-luminosity monitor" at LHC ⇒ normalize other cross sections to this, small experimental and theory errors

Dittmar, Pauss, Zurcher hep-ex/9705004

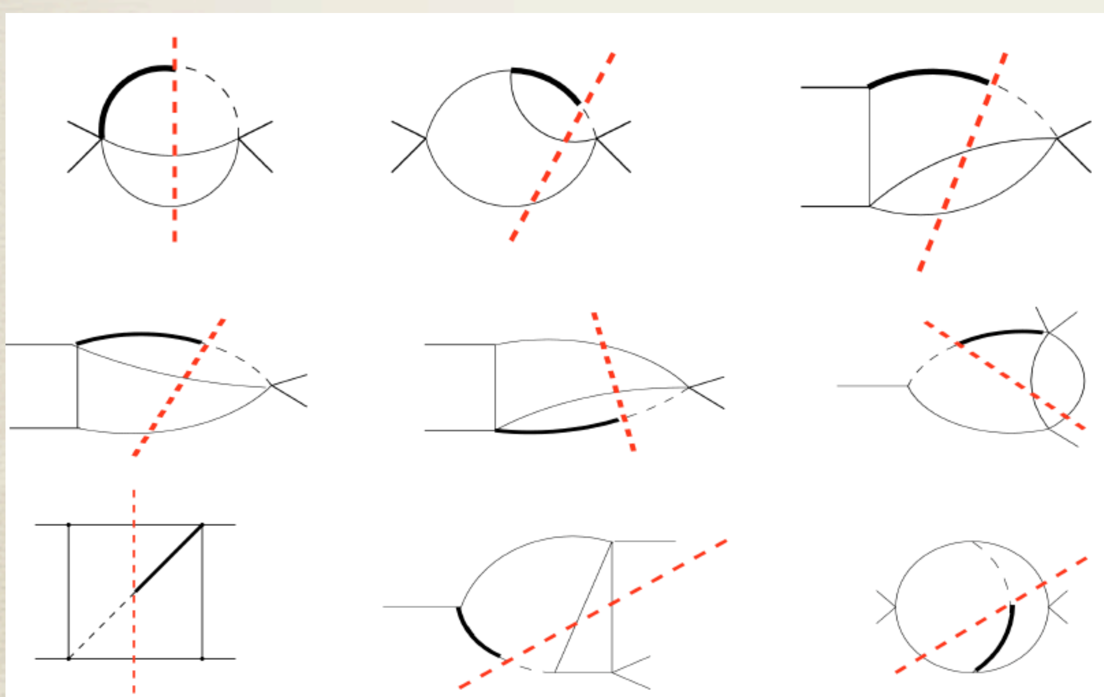
Antenna subtraction at NNLO

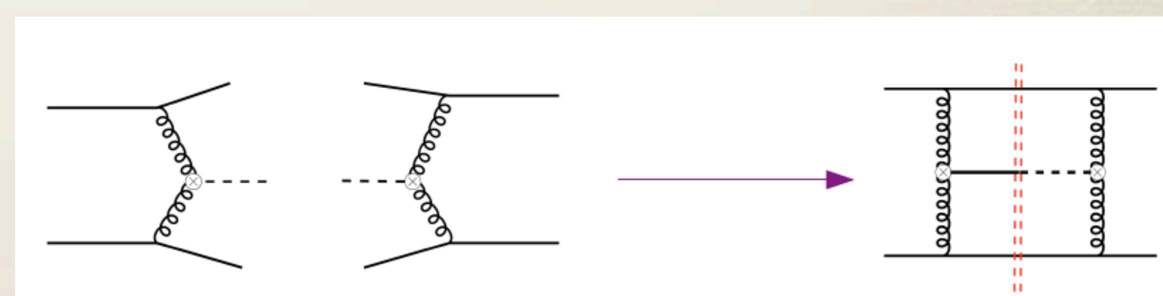
- Progress towards constructing subtraction technique for 2→2 processes at the LHC Boughezal, Gehrmann-De Ridder, Ritzmann 1001.2396



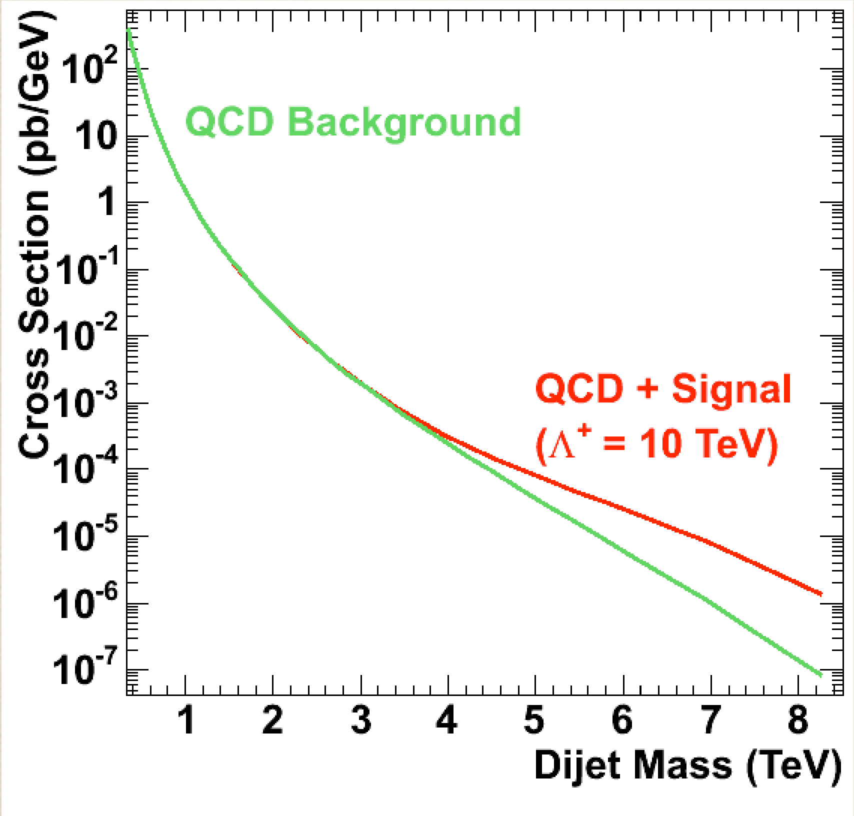
$$d\,\phi_{m+2}(k_1,\dots,k_{m+2};\,p_1,p_2) = d\,\phi_m(\tilde{k}_1,\dots,\tilde{k}_i,\tilde{k}_l,\dots,\tilde{k}_{m+2};\,x_1 p_1, x_2 p_2)$$
$$[dk_j][dk_k]\,d x_1\,d x_2\,J\,\delta(q^2 - x_1 x_2 s_{12})\,\delta(2(x_2 p_2 - x_1 p_1)\cdot q)$$

Then integrate over unresolved PS using IBP technology





Conclusions



from T. LeCompte, CTEQ 2007 summer school

- Can understand a lot by considering IR singularities of QCD amplitudes: leads to parton shower, jet definitions, ...
- Serious quantitative predictions at LHC require NLO; multi-leg methods have seen revolutionary advances, very active area!
- Effective field theory methods can simplify calculations with multiple scales
- Techniques exist for merging LO/NLO+PS
- NNLO needed for W, Z, H, PDFs+α_s, jet energy scale, top and V pairs; active area!
- Remember PDF errors *only* reflect experimental errors on used data sets!

Backup: Effective Field Theory for the Higgs

Decoupling constants

- Green's functions in full and effective theories must match in the $m_t \to \infty$ limit.

primes denote effective theory quantities

$$g_s^{0,\prime} = \zeta_g^0 g_s^0 \,, \qquad m_q^{0,\prime} = \zeta_m^0 m_q^0 \,, \qquad \xi^{0,\prime} - 1 = \zeta_3^0 (\xi^0 - 1) \,,$$

$$\psi_q^{0,\prime} = \sqrt{\zeta_2^0}\psi_q^0 \,, \qquad G_\mu^{0,\prime,a} = \sqrt{\zeta_3^0} G_\mu^{0,a} \,, \qquad c^{0,\prime,a} = \sqrt{\tilde{\zeta}_3^0} c^{0,a} \,,$$

$$\mathcal{L}_{\text{eff}}^{\text{QCD}}(g_s^0, m_q^0, \xi^0; \psi_q^0, G_\mu^{0,a}, c^{0,a}; \zeta_i^0) \;=\; \mathcal{L}^{\text{QCD}}(g_s^{0,\prime}, m_q^{0,\prime}, \xi^{0,\prime}; \psi_q^{0,\prime}, G_\mu^{0,\prime,a}, c^{0,\prime,a})$$

useful review: M. Steinhauser, hep-ph/0201075

- Equate Green's functions to derive decoupling constants (*matching calculation*)

$$\begin{aligned}
\frac{\delta^{ab}\left(-g_{\mu\nu} + \frac{p^\mu p^\nu}{p^2}\right)}{-p^2\left(1 + \Pi_G^0(p^2)\right)} &= i \int \mathrm{d}^4x e^{ipx} \langle T G_\mu^{0,a}(x) G_\nu^{0,b}(0) \rangle \\
&= \frac{1}{\zeta_3^0} i \int \mathrm{d}^4x e^{ipx} \langle T G_\mu^{0,\prime,a}(x) G_\nu^{0,\prime,b}(0) \rangle \\
&= \frac{1}{\zeta_3^0} \frac{\delta^{ab}\left(-g_{\mu\nu} + \frac{p^\mu p^\nu}{p^2}\right)}{-p^2\left(1 + \Pi_G^{0\prime}(p^2)\right)} \,.
\end{aligned}$$

EFT calculations in dim. reg. usually scaleless ⇒ vanishes

Simplify by doing matching calculation at $p^2=0$

$$\zeta_3^0 \;=\; 1 + \Pi_G^{0,h}(0)$$

⇒ just tadpole diagrams depending on m_t

Wilson coefficient derivation

- For the physical amplitude gg→h, only one operator contributes:

Wilson coefficient

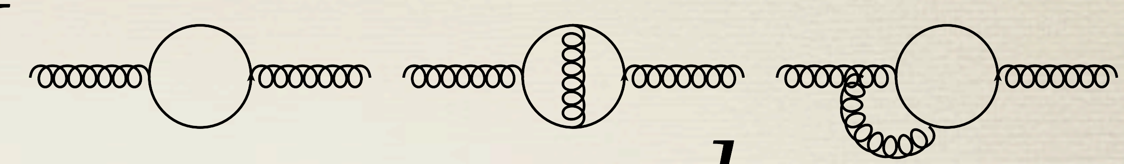
$$\mathcal{L}_{eff}^{Higgs} = -\frac{h}{v} C_1^0 \mathcal{O}_1^0$$
$$\mathcal{O}_1^0 = (G_{\mu\nu}^{0,',a})^2$$

- Do the matching calculation between full/EFT

$$\begin{aligned}
\Gamma_{\mu\nu}^{0,full}(p_1,p_2)\delta^{ab} &= i\int d^4x d^4y\, e^{ip_1\cdot x} e^{ip_2\cdot y} \langle \mathrm{T}\left[G_\mu^{0,a}(x) G_\mu^{0,b}(y) h(0)\right]\rangle \\
&= \frac{i}{\xi_3^0}\int d^4x d^4y\, e^{ip_1\cdot x} e^{ip_2\cdot y} \langle \mathrm{T}\left[G_\mu^{\prime,0,a}(x) G_\mu^{\prime,0,b}(y) h(0)\right]\rangle \\
&= i\,\xi_3^0 \int d^4x d^4y\, e^{ip_1\cdot x} e^{ip_2\cdot y} \langle \mathrm{T}\left[G_\mu^{\prime,0,a}(x) G_\mu^{\prime,0,b}(y) h(0)\right]\rangle_{1PI} \\
&= -4\,\frac{\xi_3^0 C_1^0}{v}\left[g_{\mu\nu} p_1\cdot p_2 - p_1^\nu p_2^\mu\right]\delta^{ab} \\
\frac{\xi_3^0 C_1^0}{v} &= -\frac{1}{4}\left(\frac{p_1\cdot p_2 g^{\mu\nu} - p_1^\mu p_2^\nu - p_1^\nu p_2^\mu}{(d-2)(p_1\cdot p_2)^2}\Gamma_{\mu\nu}^{0,full}(p_1,p_2)\right)_{p_1=p_2=0}
\end{aligned}$$

(for Higgs, a more elegant way to derive based on low energy theorems; see Kniehl, Spira Z. Phys. C 69 (1995))

Calculational steps

- Get the decoupling constant $(\xi_3)^0$: $\Big[$  $+\ldots \Big]_{p_1=p_2=0}$

$$
\begin{aligned}
\Pi_G^t(0) &= \frac{\alpha_s^0}{\pi}\left[\frac{1}{6\epsilon} - \frac{1}{6}\ln\frac{(m_t^0)^2}{\mu^2} + \epsilon\left(\frac{\pi^2}{72} + \frac{1}{12}\ln^2\frac{(m_t^0)^2}{\mu^2}\right)\right] \\
&+ \left(\frac{\alpha_s^0}{\pi}\right)^2\left[\frac{3}{32\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{1}{64} - \frac{3}{16}\ln\frac{(m_t^0)^2}{\mu^2}\right) + \frac{91}{1152} + \frac{\pi^2}{64} + \frac{3}{16}\ln^2\frac{(m_t^0)^2}{\mu^2}\right]
\end{aligned}
$$

- Replace bare quantities with renormalized ones:

$$
\begin{aligned}
\alpha_s^0 &= Z_g^2\alpha_s^{(6)} \\
m_t^0 &= Z_m m_t
\end{aligned}
$$

- Use decoupling constants to go to a theory with five active flavors

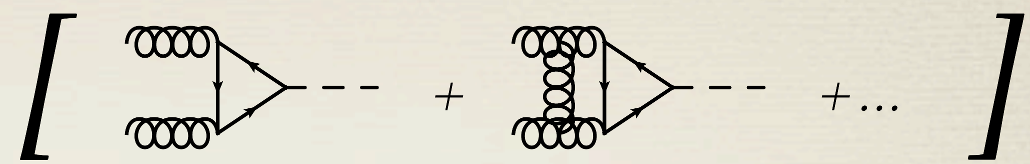
Derive coupling constant
from gluon-ghost vertex:

$$
\left[1 + \Gamma^0_{G\bar{c}c}(p,k)\right] = \zeta_g^0\tilde{\zeta}_3^0\sqrt{\zeta_3^0}\left[1 + \Gamma^{0\prime}_{G\bar{c}c}(p,k)\right]
$$

$$
\tilde{\zeta}_3^0 = 1 + \Pi_c^{0,h}(0)
$$

$$
\begin{aligned}
\alpha_s^{(5)} &= \xi_g^2\alpha_s^{(6)} \\
\xi_g &= 1 + \frac{\alpha_s^{(6)}}{\pi}\frac{1}{12}\ln\frac{m_t^2}{\mu^2}
\end{aligned}
$$

Calculational steps

- Get the bare Wilson coefficient:  $_{p_1=p_2=0}$

$$\xi_3^0 \, C_1^0 = \frac{\alpha_s^0}{\pi}\left\{-\frac{1}{3} + \epsilon\frac{1}{3}\ln\frac{(m_t^0)^2}{\mu^2}\right\} - \frac{1}{4}\left(\frac{\alpha_s^0}{\pi}\right)^2$$

- Operator O_1 requires renormalization, RG invariance gives inverse renormalization to C_1; since it is $(G_{\mu\nu})^2$, clearly connected to the beta-function.

$$\begin{aligned} C_1 &= \frac{1}{Z_{11}} C_1^0 \\ \frac{1}{Z_{11}} &= 1 + \frac{\alpha_s^{(5)}}{\pi}\frac{\beta_0}{\epsilon} + \left(\frac{\alpha_s^{(5)}}{\pi}\right)^2 \frac{\beta_1}{\epsilon} \end{aligned}$$

- Combine all pieces, arrive at the final result.

$$C_1 = -\frac{1}{3}\frac{\alpha_s^{(5)}}{\pi} - \frac{11}{12}\left(\frac{\alpha_s^{(5)}}{\pi}\right)^2$$

Integration-by-parts

- IBP identities for the calculation of the bare Wilson coefficient:

$$
\begin{aligned}
\mathrm{Tri}(\nu_1,\nu_2,\nu_3) &= \int d^dk_1 d^dk_2 \frac{1}{[k_1^2-m_t^2]^{\nu_1}[k_2^2-m_t^2]^{\nu_2}[(k_1-k_2)^2]^{\nu_3}} \\
0 &= (-\nu_3-2\nu_1+d)\,\mathrm{Tri}(\nu_1,\nu_2,\nu_3) - \mathrm{Tri}(-1+\nu_1,\nu_2,1+\nu_3)\nu_3 \\
&- 2\,\mathrm{Tri}(1+\nu_1,\nu_2,\nu_3)\,m_t^2\,\nu_1 + \mathrm{Tri}(\nu_1,-1+\nu_2,1+\nu_3)\,\nu_3 \\
0 &= (\nu_3-\nu_2)\,\mathrm{Tri}(\nu_1,\nu_2,\nu_3) - \mathrm{Tri}(-1+\nu_1,1+\nu_2,\nu_3)\,\nu_2 + \mathrm{Tri}(-1+\nu_1,\nu_2,1+\nu_3)\,\nu_3 \\
&- \mathrm{Tri}(\nu_1,-1+\nu_2,1+\nu_3)\,\nu_3 + \mathrm{Tri}(\nu_1,1+\nu_2,-1+\nu_3)\,\nu_2 - 2\,\mathrm{Tri}(\nu_1,1+\nu_2,\nu_3)\,m_t^2\,\nu_2 \\
0 &= (\nu_3-\nu_1)\,\mathrm{Tri}(\nu_1,\nu_2,\nu_3) - \mathrm{Tri}(-1+\nu_1,\nu_2,1+\nu_3)\,\nu_3 - \mathrm{Tri}(1+\nu_1,-1+\nu_2,\nu_3)\,\nu_1 \\
&+ \mathrm{Tri}(1+\nu_1,\nu_2,-1+\nu_3)\,\nu_1 + \mathrm{Tri}(\nu_1,-1+\nu_2,1+\nu_3)\,\nu_3 - 2\,\mathrm{Tri}(1+\nu_1,\nu_2,\nu_3)\,m_t^2\,\nu_1 \\
0 &= (-\nu_3-2\nu_2+d)\,\mathrm{Tri}(\nu_1,\nu_2,\nu_3) + \mathrm{Tri}(-1+\nu_1,\nu_2,1+\nu_3)\,\nu_3 \\
&- \mathrm{Tri}(\nu_1,-1+\nu_2,1+\nu_3)\,\nu_3 - 2\,\mathrm{Tri}(\nu_1,1+\nu_2,\nu_3)\,m_t^2\,\nu_2
\end{aligned}
$$

- Can solve via Gaussian elimination (S. Laporta 2000), for which implemented algorithms exist (Anastasiou, Lazopoulos 2004)
- One MI exists: Tri(1,1,0)

$$\mathrm{Tri}(2,3,2) = \mathrm{Tri}(1,1,0)\,\frac{(-2+d)\,(d-4)\,(-6+d)^2}{64\,m_t^{10}(-7+d)},\ \text{etc.}$$